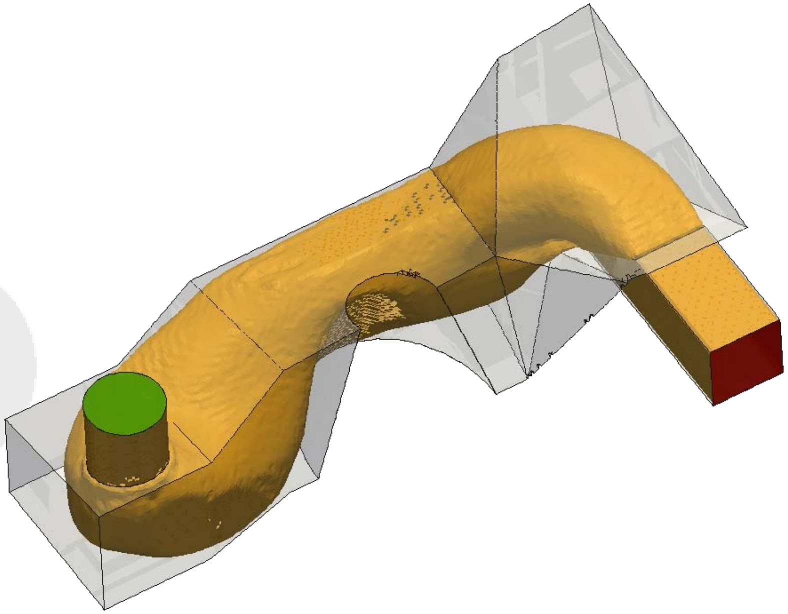
# “LEVEL-SET BASED TOPOLOGY OPTIMIZATION USING THE CONTINUOUS ADJOINT METHOD ”

*OPT-i*

*4th-6th June 2014*

*Kos Island, Greece*



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# AboutFLOW Project

- Adjoint-Based optimization of industrial and unsteady flows
- http://aboutflow.sems.qmul.ac.uk

"This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement no [317006]".

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  - Level – Set Method
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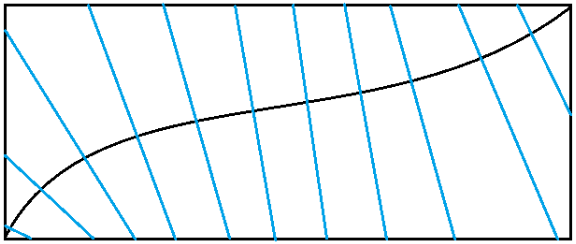
# Theory | Level – Set Method

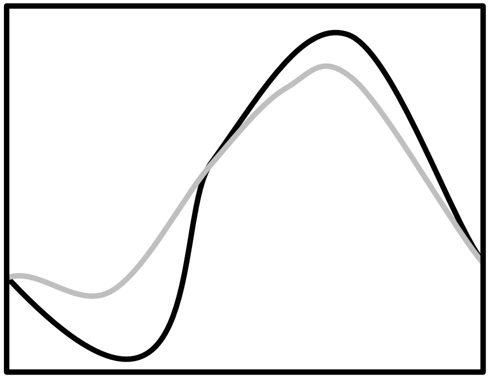
- Inspired from a paper of J.A. Sethian*
- Design variables (ϕ) are the signed distances from the interface.
- Constitutes three parts:
  - Velocity extension – velocity extended towards the normal direction of the interface

$$\frac{\partial (W_i G_s)}{\partial x_i} - \frac{\partial W_i}{\partial x_i} G_s - k_{Lap} \frac{\partial^2 G_s}{\partial x_i^2} = 0 \quad W_i = sigh(\varphi) \frac{\frac{\partial \varphi}{\partial x_i}}{\left| \frac{\partial \varphi}{\partial x_i} \right|}$$



  - Evolution – move the interface (update the ϕ field) by solving a transport equation

$$\frac{\partial \varphi}{\partial t} + \frac{\partial (G_i \varphi)}{\partial x_i} - \varphi \frac{\partial G_i}{\partial x_i} = 0$$



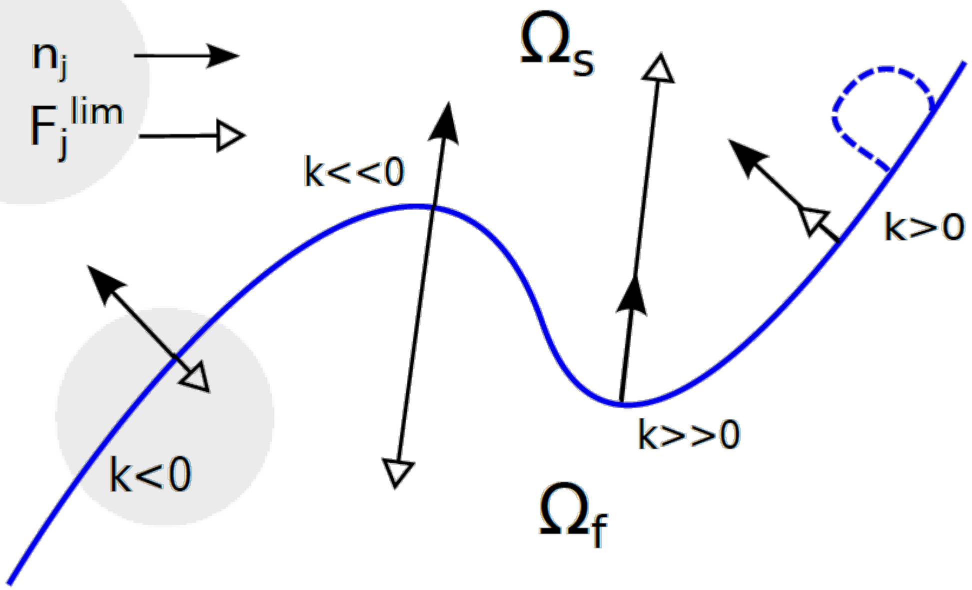
*J.A. Sethian. Evolution, implementation, and application of level set and fast marching methods for advancing fronts. Journal of Computational Physics, 169:503–555, 2001.*

# Theory | Level – Set Method

- Reinitialization – the new ϕ field has to be corrected
  - Narrow band approach
  - $\frac{\partial \varphi}{\partial t} + W_i \frac{\partial \varphi}{\partial x_i} = sign(\varphi)$ , t pseudo time

- Curvature calculation

$$F_j^{lim} = kn_j = \frac{\partial^2 \varphi}{\partial x_i^2} \frac{\partial \varphi}{\partial x_j}$$



# Theory | Continuous Adjoint Method

- Cost doesn't increase with the number of parameters
- The calculation of the sensitivity derivatives is approximately equivalent with the solution of one primal problem
- Avoid calculating the computationally expensive terms by making their multipliers zero
- After the derivation of the augmented objective function $F_{aug}$
  - $F_{aug} = F + \int_{\Omega} qR^p d\Omega + \int_{\Omega} u_i R_i^v d\Omega$
  - $\frac{\delta F_{aug}}{\delta b} = \frac{\delta F}{\delta b} + \int_{\Omega} q \frac{\partial R^p}{\partial b} d\Omega + \int_{\Omega} u_i \frac{\partial R_i^v}{\partial b} d\Omega + \int_{\Omega} (u_i R_i^v + qR^p) \frac{\delta x_k}{\delta b} n_k dS = \ldots\ldots\ldots \Rightarrow$

$$\Rightarrow G = \frac{\delta F_{aug}}{\delta b} = \int_{\Omega} (R^q) \frac{\partial p}{\partial b} d\Omega + \int_{\Omega} (R_i^u) \frac{\partial v_i}{\partial b} d\Omega + \int_{S} (BC^1) \frac{\partial p}{\partial b} dS + \int_{S} (BC^2) \frac{\partial v_i}{\partial b} dS + \int_{\Omega} (G_{\Omega}) d\Omega + \int_{S} (G_S) dS$$

- $R^q$: adjoint pressure equation
- $R^u$: adjoint velocity equations
- BCs: adjoint boundary conditions
- $G_s$: surface sensitivities
- $G_{\Omega}$: volumetric sensitivities
- F: objective function

# Theory | Continuous Adjoint Method

- Primal equations

$$R^p = \frac{\partial v_j}{\partial x_j} = 0$$

$$R_i^v = v_j \frac{\partial v_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_j}\left[(\nu + \nu_t)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right]$$

$$R_z = f(z) = Convection + Diffussion + Production + Dissipation = 0$$

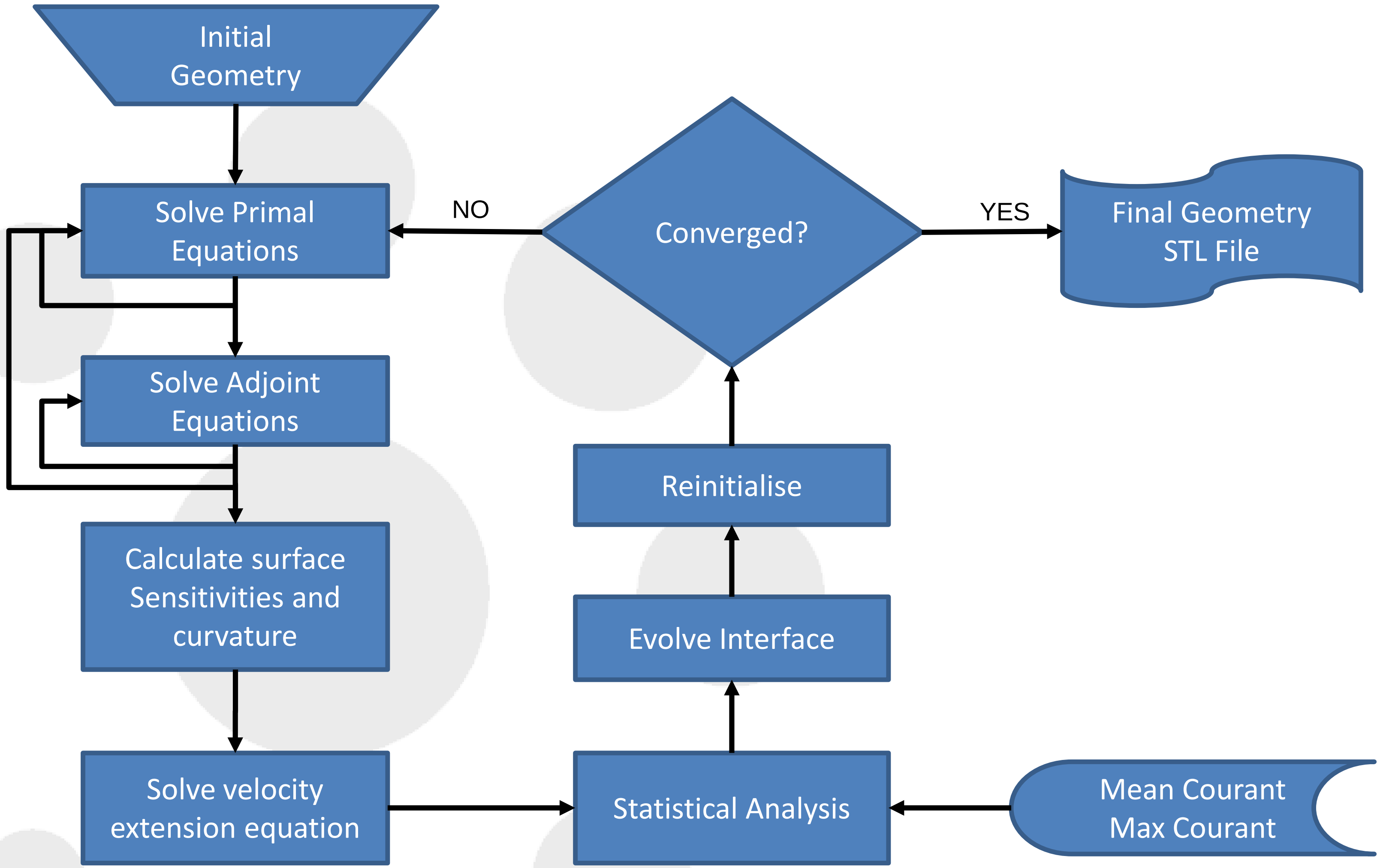
- Adjoint equations

$$R_q = \frac{\partial u_i}{\partial x_i} = 0$$

$$R_i^v = -v_j\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{\partial q}{\partial x_i} - \frac{\partial}{\partial x_j}\left[(\nu + \nu_t)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] = 0$$

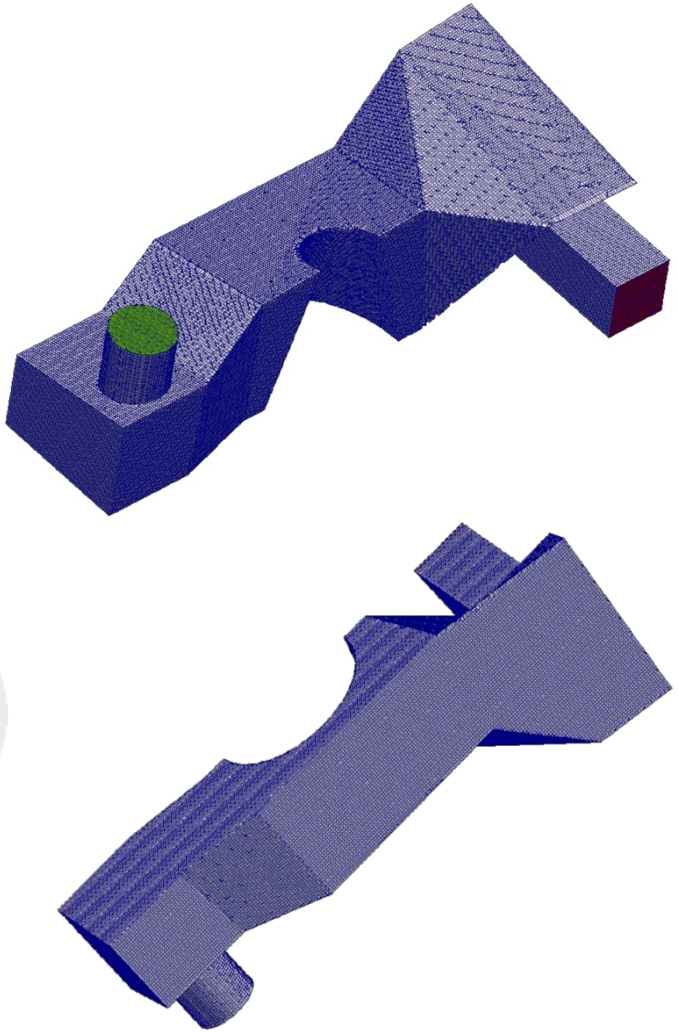
$$G_j = (\nu + \nu_t)\left(\frac{\partial u_i}{\partial n} + \frac{\partial u_{\langle n \rangle}}{\partial x_i}\right)\frac{\partial v_i}{\partial n} S_j$$

# Theory | General Algorithm



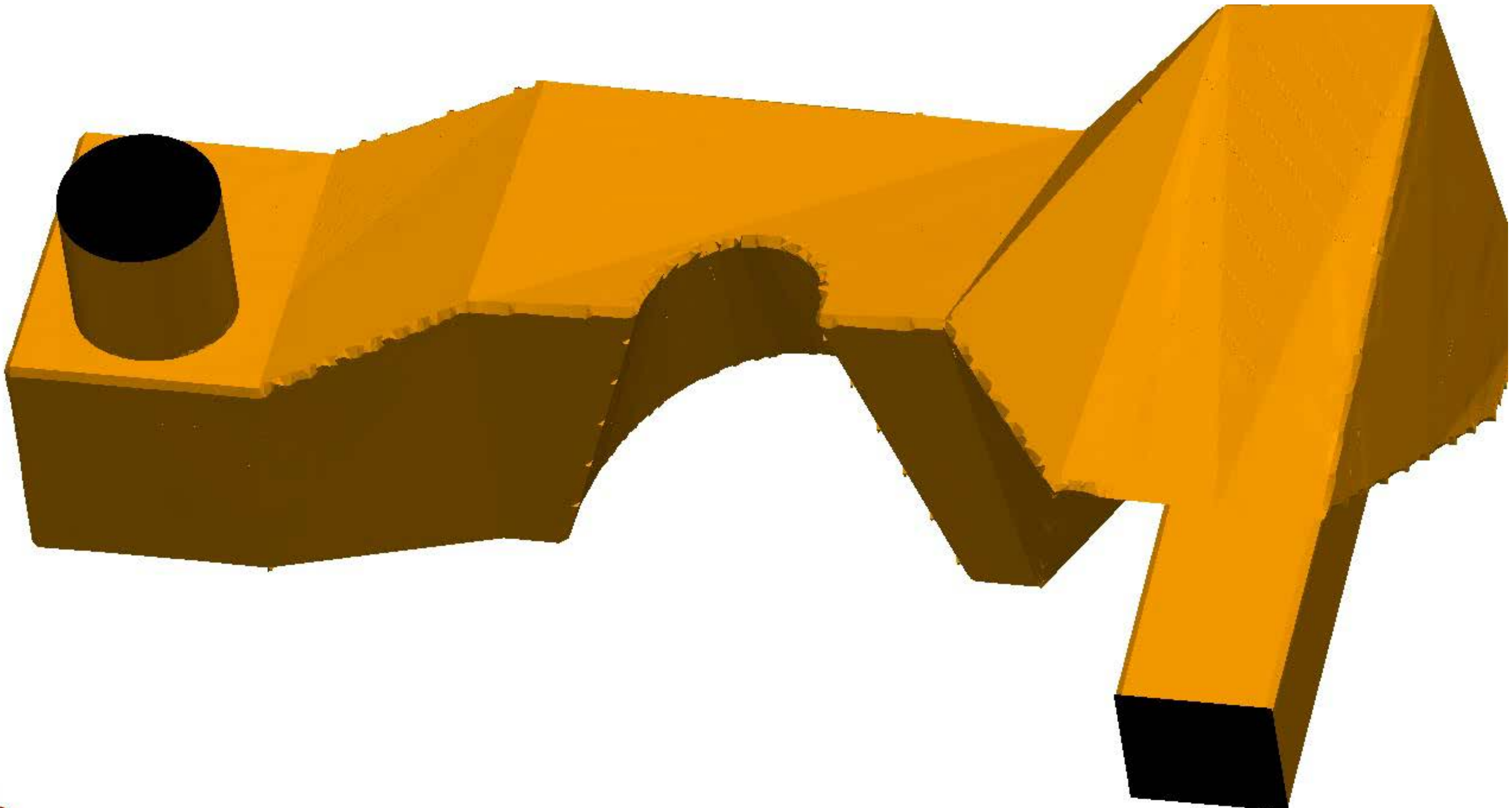
# HVAC Duct | Case description

- Mesh: 600.694 cells
- Inlet: Red patch
- Outlet: Green patch
- Fluid: Air
- Boundary conditions:
  - Inlet: $\dot{m} = 0.0133$ kg/s
  - Outlet: $p = 0$ Pa
- Objective: minimization of power losses

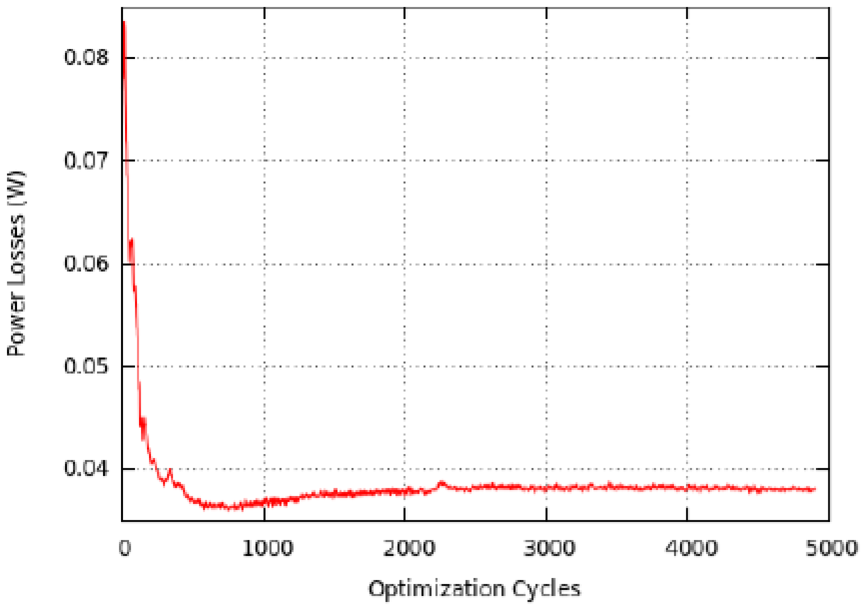


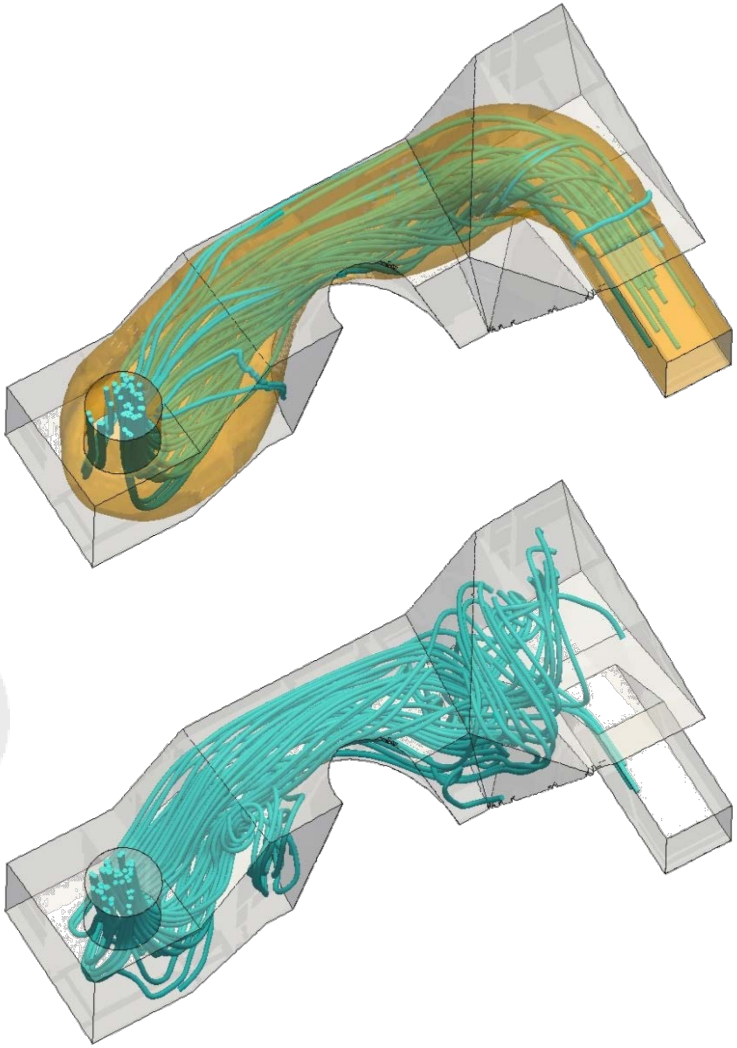
# HVAC Duct | Results

- Yellow surface: zero level-set contour



# HVAC Duct | Results



| Geometry | Power Losses (W) |
|---|---|
| Baseline | 0.077 W |
| Optimized | 0.038 W |
| Percentage | 50.65 % |





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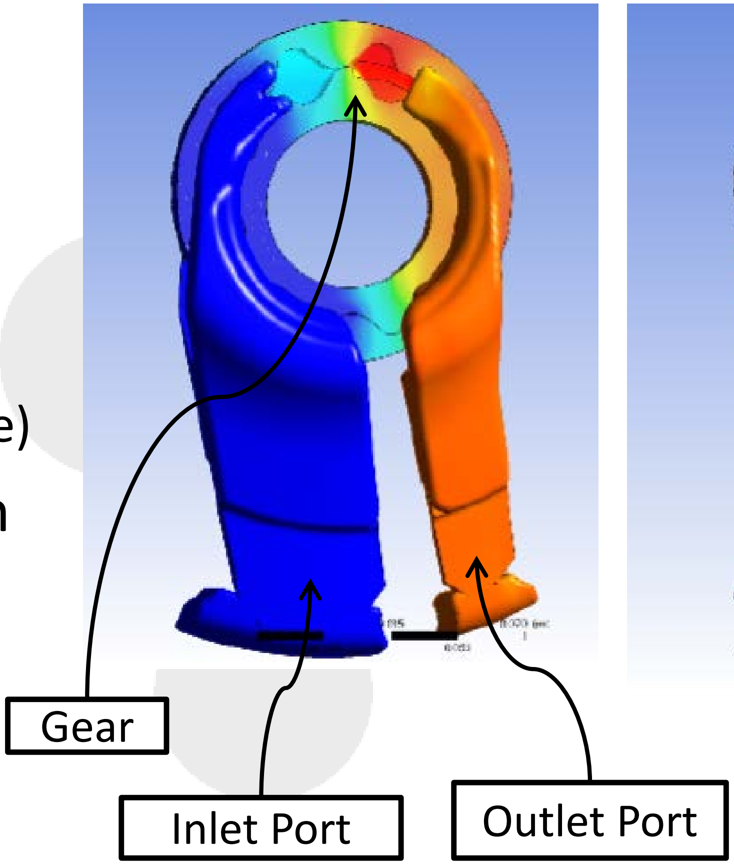
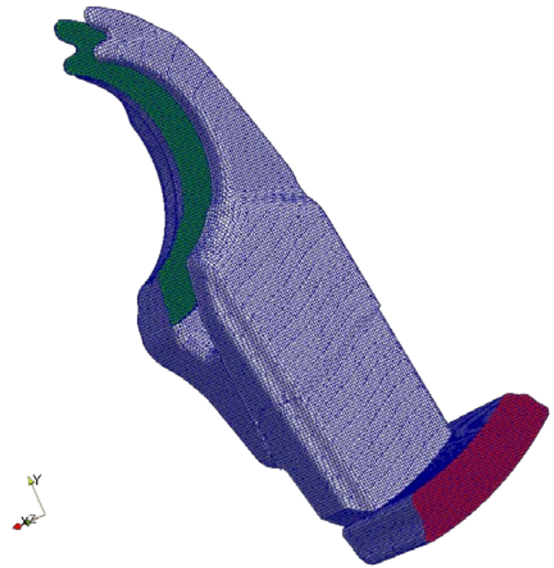
# Gear Pump | Case description

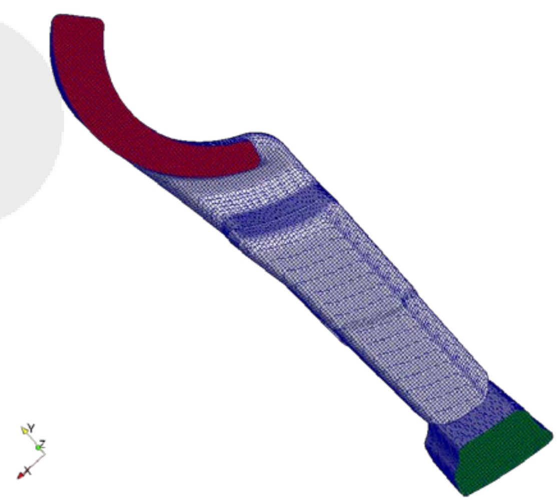
- Geometry provided by Aisin AW
- Case separated to two parts:
  - Inlet port (low pressure)
  - Outlet port (high pressure)
- Objective: minimization of power losses




# Gear Pump | Case description

- Fluid: lubricant oil
  - V = 1.0905 $10^{-5}$ $m^2/s$
  - $\rho$ = 829 $kg/m^3$
- Boundary conditions:
  - Inlet port:
    - Inlet: p = 0 Pa
    - Each outlet: $\dot{m}$ = 0.375 kg/s
  - Outlet port:
    - Inlet: $\dot{m}$ = 0.75 kg/s
    - Outlet : p = 500 kPa
- Mesh size:
  - Inlet port: 330.000 cells
  - Outlet port: 175.000 cells









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# Gear Pump | Results

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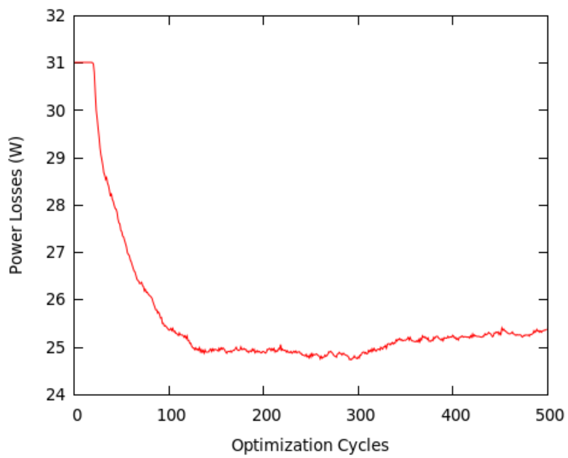
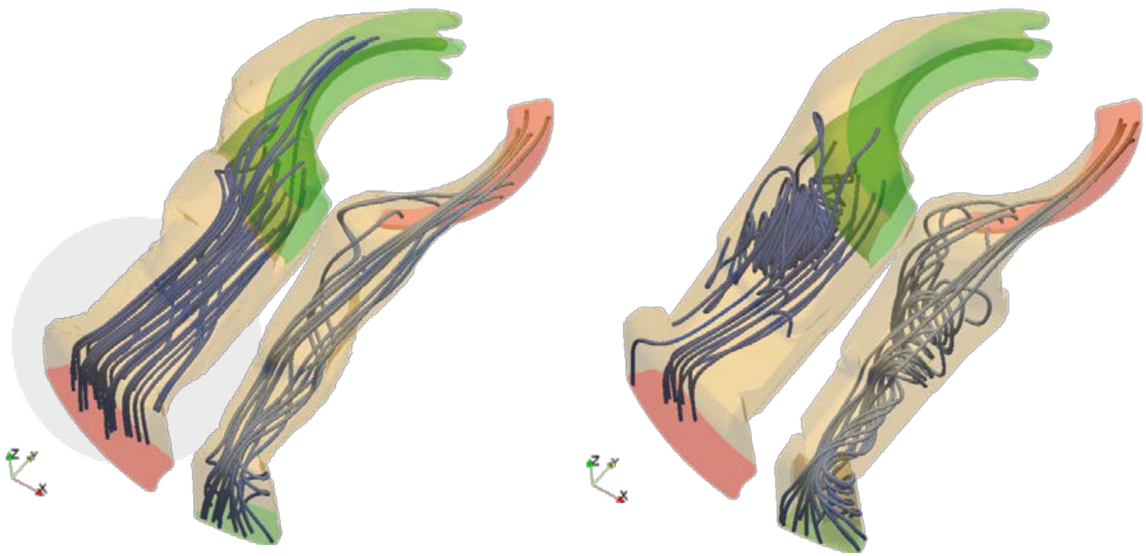
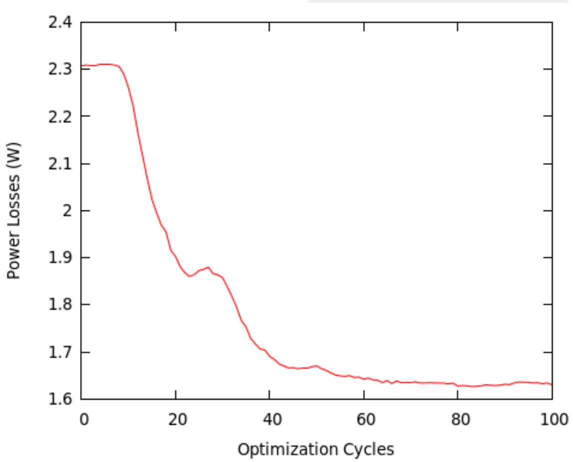





| Power Losses | Inport | Outport | Gear Pump |
|---|---|---|---|
| Baseline | 2.208 W | 31.017 W | 33.325 W |
| Optimized | 1.635 W | 25.379 W | 27.013 W |
| Percentage | 29.17 % | 18.18 % | 18.94 % |

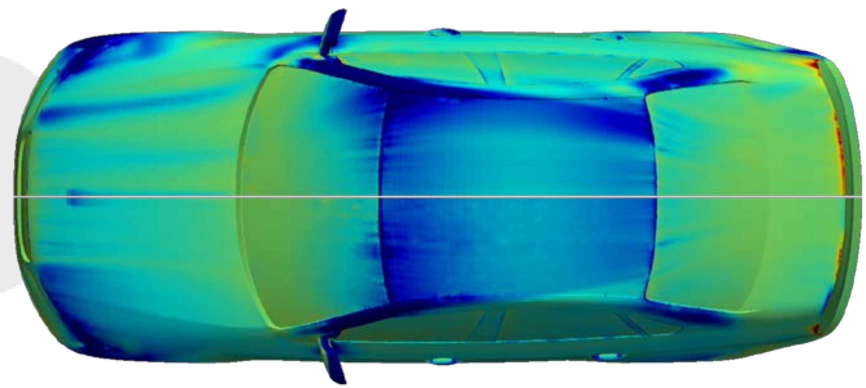
# Conclusions

- Advantages
  - Better control of the optimization
  - More manufacturable surfaces
  - Optimized geometries can be used directly from the manufacturers
  - Better accuracy
- Disadvantages
  - Level – set equations sometimes are very stiff

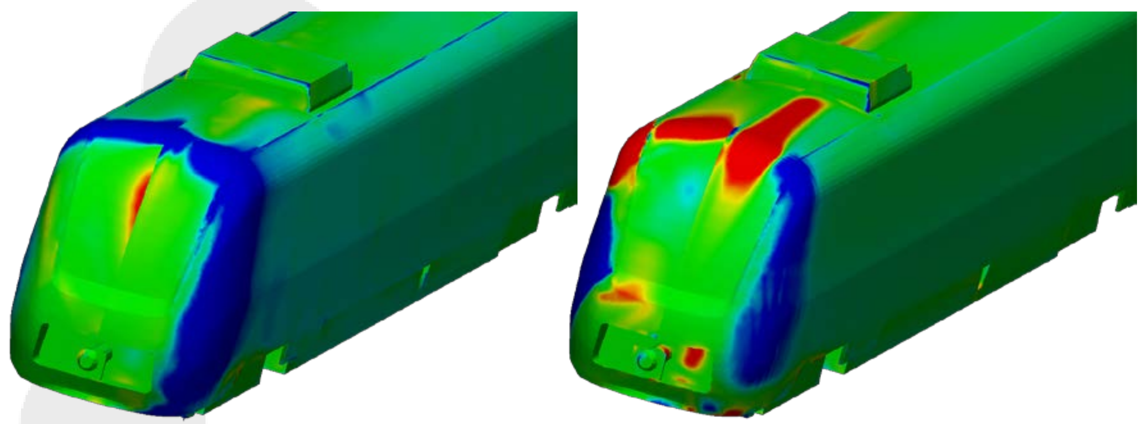
# Recent developments

- 2nd order accurate adjoint convection
- Adjoint Transpose Convection (ATC) problem

- DRIVAIR passenger car
  - 2nd order adjoint convection
  - 1st order adjoint convection + ATC off 1st cell



- Regional Train
  - Left: 1st order adjoint convection + ATC off 1st cell
  - Right: 2nd order accurate adjoint convection



# The end

**Thanks for your time**

**Any questions?**