Output-based r-refinement using a flow-coupled system solve.

Mateusz Gugala* and Jens-Dominik Müller†
Queen Mary University of London, London, E1 4NS, United Kingdom

I. Introduction

The adjoint method[1] is well-established as the most efficient method for aerodynamic shape optimisation with CFD. The adjoint solution quantifies the linearised effect of a unit source term in the conservation equation in each mesh point. If the local error is expressed as a source term, then the adjoint can be also used to drive a solution-adaptive mesh refinement process that computes a more accurate objective function at lower computational cost compared to error-estimators without adjoint weighting or compared to heuristic sensors. The adjoint-weighted truncation error known also as an output-based indicator gained popularity as an effective driver for an adaptation process - see e.g.[2–4] Through the adjoint weighting, the mesh adaptation is very effectively targeted to those areas of the computational domain where the objective function is highly sensitive to local error. This is the key advantage of an output-based indicator as compared to other approaches such as e.g. gradient/Hessian-based sensors or unweighted truncation-error-based sensors. While the latter at least attempts to estimate the actual errors, both of these methods apply the refinement to all errors, regardless of whether they are relevant to the computation of the objective function or not.

The adjoint-weighted adaptation sensor can be used as a driver for the refinement process in various ways, among which the three main techniques are: h-refinement, $r-$refinement and p-refinement, in this work we focus on $r-$refinement, of which several examples can be found in the literature. Dwight[5] proposed using linear elasticity mesh deformation in order to cluster the nodes in the areas with highest sensor values, which can be achieved by applying a non-zero body force term. Tyson et al.[6] compared several methods based on the fundamental principle of equi-distributing of a weight function across the domain where the weight function is some measure of the local error. However, these methods do not connect the system of flow equations and their discretisation to the deformation methodology which can result in non-optimal distribution of computational points for a general case.

In this work, we present an alternative methodology for $r-$refinement which couples deformation algorithm with the system of flow equations, with the key goal to obtain an optimal node distribution and mesh quality for the given error estimate, discretisation scheme, and flow solution. The cube case with a 3D manufactured solution is used for testing purposes and later the technique is applied to the Onera M6 wing case.

II. A flow-coupled r-refinement methodology

The output error (OE) provides the information on how the truncation errors (TE) in each control volume and for each equation contribute to the error in the output of interest e.g. lift or drag. More formally it can be derived as the adjoint-weighted truncation error presented in equation (1). The subscript h stands for the discrete space with characteristic mesh size h.

$$OE_h = v_h^T TE_h. \tag{1}$$

The truncation error estimate i.e. TE_h is obtained using a multi-grid approach between a fine mesh h and a topologically inconsistent coarse mesh H

$$TE_h = -\mathcal{I}_H^h R_H(\mathcal{I}_h^H U_h), \tag{2}$$

*PhD candidate, School for Engineering and Materials Science, m.gugala@qmul.ac.uk
†Senior lecturer, School for Engineering and Materials Science, j.mueller@qmul.ac.uk

where the operator $\mathcal{I}$ stands for inter-grid interpolation. The complete methodology and derivations are presented by the author in.[7]

For the system of flow equations $R_h(U_h, X_h)$ the Jacobian matrix $\left(\frac{\partial R}{\partial X}\right)_h$ relates the truncation error with the nodal positions X, thus when displacing the nodes in the negative gradient direction the truncation errors will decrease. The deformation algorithm based on the least squares minimisation problem formed using the output error (OE_h) as a driving term, and the Jacobian $\left(\frac{\partial R}{\partial X}\right)_h$ as a system matrix is derived as presented in Eq. (3).

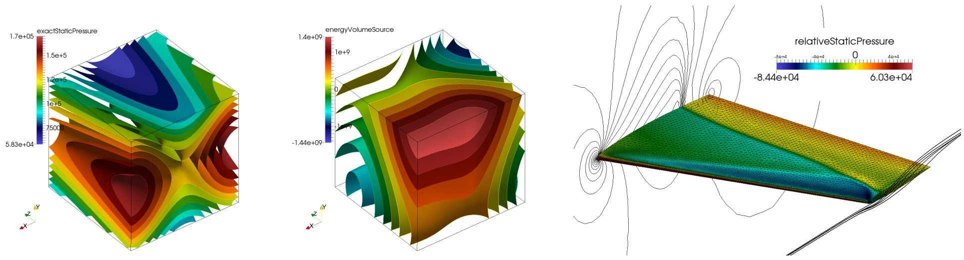
$$\min_x \left|\left| \ \left(\frac{\partial R}{\partial X}\right)_h \delta X_h - OE_h \ \right|\right|_2 \tag{3}$$

The Jacobian $\frac{\partial R}{\partial X}$ is derived using the AD tool Tapenade[8] applied to the discretised system $R_h(U_h, X_h)$ with the vector X_h as an independent variable. As a result of strong coupling of the formulated least square system (3) with discretised flow equations, the information on the error directionality, not only magnitude should be obtained, thus leading to optimal nodes displacement.

The in-house flow and adjoint solver mgopt developed at Queen Mary University of London is used in this work. It is a finite-volume, compressible, vertex-centred code with an edge-based data structure.

III. Results

The cube case with a 3-D manufactured solution by Roy[9] is used for verification of the proposed $r-$refinement strategy. The flow is governed by the compressible, supersonic Euler equ., Fig. 1 shows an example pressure field and corresponding manufactured source term. The Onera M6 wing[a] is used as a more realistic and physically meaningful application example - Fig. 1c.



(a) *pressure (p)* (b) *energy src (f_e)* (c) *M6 wing Transonic flow and mesh.*

Figure 1: The 3D supersonic manufactured solution.

Acknowledgements

This work has been conducted within the About Flow project on Adjoint-based optimisation of industrial and unsteady flows, which has received funding from the European Unions Seventh Framework Programme for research, technological development and demonstration, Grant Agreement No. 317006.

References

[1]Pironneau, O., "On Optimum Design in Fluid Mechanics," Vol. 64, 1974, pp. 97–110.

[2]Fidkowski, K. J. and Darmofal, D. L., "Review of Output-Based Error Estimation and Mesh Adaptation in Computational Fluid Dynamics," Vol. 49, No. 4, 2011, pp. 673–694.

[3]Darmofal, D. and Venditti, D., "Anisotropic grid adaptation for functional outputs: application to two-dimensional viscous flows," *JCP*, Vol. 187, 2003, pp. 22–46.

[4]Ponsin, J., Fraysse, F., Gómez, M., and Cordero-Gracia, M., "An adjoint-truncation error based approach for goal-oriented mesh adaptation," *Aerospace Science and Technology*, Vol. 41, 2015, pp. 229–240.

[5]Dwight, R. P., "Robust mesh deformation using the linear elasticity equations," *Computational Fluid Dynamics 2006*, Springer, 2009, pp. 401–406.

[6]Tyson, W. C., Derlaga, J. M., Roy, C. J., and Choudhary, A., "Comparison of r-Adaptation Techniques for 2-D CFD Applications," *AIAA Paper*, Vol. 2611, 2015, pp. 22–26.

[7]Gugala, M. and Müller, J.-D., "Towards an output-based re-meshing for trubomachinery applications," *Proceedings ECCOMAS 2016 (in revision)*.

[8]Hascoët, L. and Pascual, V., "TAPENADE 2.1 user's guide," Technical Report 0300, INRIA, 2004.

[9]Roy, C. J., Smith, T. M., and Ober, C. C., "Verification of a compressible CFD code using the method of manufactured solutions," *energy*, Vol. 2, 2002, pp. s2.

[a]http://www.grc.nasa.gov/WWW/wind/valid/m6wing/m6wing.html