Incorporation of Bird Strike Requirements in MDO of an Aircraft Wing using Sub-space Metamodels

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A metamodel based multidisciplinary design optimisation of a conceptual aircraft wing model is presented. The disciplines considered are bird impact at a number of critical locations along the leading edge as well as static bending and twisting stiffness of the wing. The bird strike simulations are many times more costly in terms of computational budget than the the static load cases and as 100 sizing design variables are considered the problem may become very expensive. The multidisciplinary design optimisation is carried out using a method previously proposed by the authors for taking into account disparity in design variable dependence of the disciplines. This design variable dependence is specified by the designer and used to build metamodels in only the space of the significant variables to each discipline. This means that the number of required points for each metamodel, and the associated computational cost for their evaluation, can be reduced. The method is implemented within the optimisation framework known as the mid-range approximation method together with a recovery mechanism for erroneous identification of significant variables. It is shown that, by using the proposed approach to take into account the local design variable dependence of the individual bird strike simulations, the optimisation can be carried out to a much reduced computational budget to what would otherwise be required.

1. Introduction

This paper presents an efficient method of incorporating bird strike as well as stiffness requirements in multidisciplinary optimisation of an aircraft wing including 100 sizing design variables. Bird strike simulations are typically several times more costly than stiffness simulations and as the bird can potentially impact the wing at any location along the leading edge, one have to consider several simulations of the bird impacting critical locations in the same optimisation. Furthermore, gradients of the response functions are not available. This makes the computational cost of the bird strike requirements in the multidisciplinary optimisation problem disproportionately large compared to the stiffness requirements.

In this work, advantage is taken of the fact that each bird impact is a local event, influencing only a small part of the wing, and hence only a small number of the design variables. A method, previously proposed by the authors [1–3], for making use of disparate design variable dependence of the individual disciplines in multidisciplinary optimisation is here used to reduce the number of required evaluations, and hence the overall computational budget of the optimisation. Meta-models are built considering only a subset, of the full set of design variables, significant to the individual disciplines. The method relies on the designer to identify the significant variables for each load case through, for instance, engineering judgement or initial ranking studies. However, if such identification is erroneous a recovery mechanism, implemented as part of a trust-region strategy, is used to recover from resulting metamodelling errors by updating the values of the insignificant variables to align with the current best point at the end of each iteration.

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2. Mid-range approximation method

The mid-range approximation method (MAM), also known as the multi-point approximation method, was originally reported by [4, 5] and [6]. The MAM solves a typical constrained optimisation problem in the form:

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\
\text{subject to} \quad & f_j(\mathbf{x}) \leq 1, \qquad j = 1, \ldots, m \\
& A_i \leq x_i \leq B_i, \quad i = 1, \ldots, n
\end{aligned}
\tag{1}
$$

where $f_0(\mathbf{x})$ is the objective function, $f_j(\mathbf{x})$ is the j-th constraint, $\mathbf{x}$ is the vector of design variables and A_i and B_i are the upper and lower bounds respectively on the design variable x_i. The optimisation problem (1) is replaced by a sequence of approximate sub-problems defined as:

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & \tilde{f_0}^k(\mathbf{x}) \\
\text{subject to} \quad & \tilde{f_j}^k(\mathbf{x}) \leq 1, \qquad j = 1, \ldots, m \\
& \left.\begin{aligned} & A_i^k \leq x_i \leq B_i^k \\ & A_i^k \geq A_i \\ & B_i^k \leq B_i \end{aligned}\right\} \quad i = 1, \ldots, n
\end{aligned}
\tag{2}
$$

where k denotes the current iteration number. $\tilde{f_0}^k(\mathbf{x})$ is a metamodel of the objective function and $\tilde{f_j}^k(\mathbf{x})$ is a metamodel of the j-th constraint function, both considered valid only in the current trust region. A_i^k and B_i^k are the bounds of the current trust region where the sub-problem (2) is solved for the current iteration. The solution procedure for each sub-problem consists of sampling, creating metamodels, solving the approximate optimisation problem and determining a new location and size of the trust region for the next iteration. The trust region will move and change size after each iteration until the termination criterion is reached. Figure 1 illustrates the history of trust regions through the sequence of sub problems in two dimensions. The trust region strategy has gone through several developments to account for the presence of numerical noise in the response function values [7, 8], occasional simulation failures [9], and improvements for high performance computing [10]. In this work a doe technique based on extensible lattice sequences [11], and a kriging metamodelling technique as outlined in [12], is used.

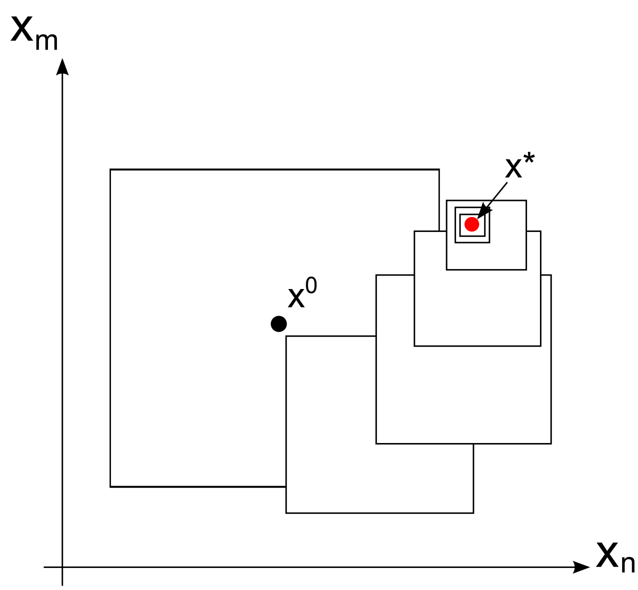


Figure 1: Typical history of the trust regions. In every iteration of the optimization process the new trust region is centered around the current solution and either kept the same size, reduced or enlarged.

3. Sub-space metamodels

Suppose that the responses belonging to a discipline in the MDO problem only depend on a subset of the full set of design variables, i.e. a set of the variables has none or very little influence on the responses of the particular discipline. An example of this from the automotive industry can be seen in Figure 2. The figure shows the results from a simulation of a vehicle subjected to a front crash load case. Each element is coloured according to its level of internal energy. It can be concluded that, as can be expected, the energy absorption is concentrated in the front of the vehicle and is not much affected by the rest of the structure. A conceptual partitioning based on design variable dependence for four common automotive disciplines, *Front Crash*, *Side Crash*, *Rear Crash* and *Noise Vibration and Harshness* (NVH) are shown in Figure 3. With this partitioning discipline-related metamodels, hereafter denoted sub-space metamodels, can be built, depending only on the set of significant variables for each discipline. This method has been presented in several previous publications, e.g. by [13], [14], [1] and [15].

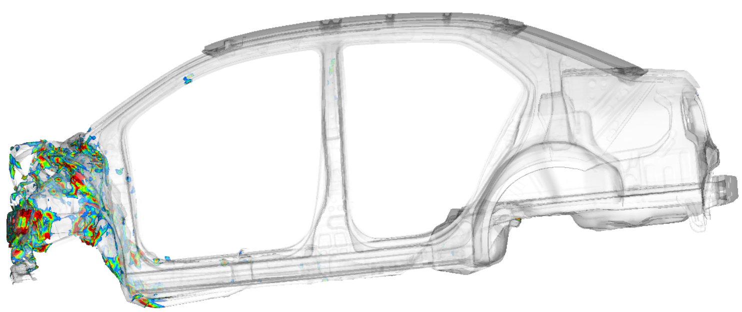


Figure 2: Front crash simulation of an automotive model. Each element is coloured according to its level of internal energy.[a]

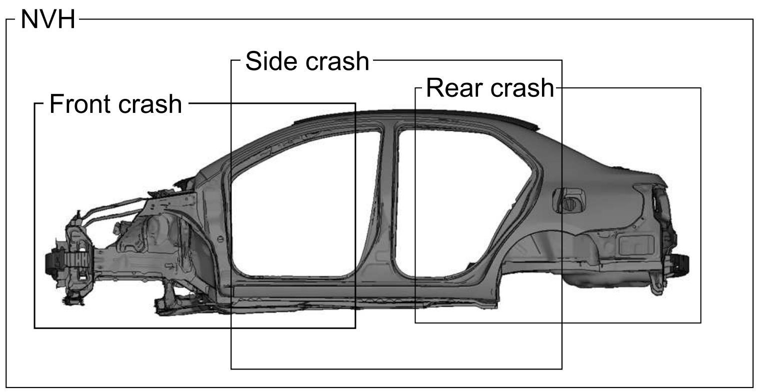


Figure 3: Conceptual partitioning of an automotive model into significant design variable sets related to each discipline

Here a mathematical formulation for introducing sub-space metamodels in metamodel assisted MDO is given. Unlike in previous work, all response functions are assumed to be defined in the full variable space of the optimisation problem in order to control insignificant variables. It will be shown that when sub-space metamodels are used within a trust region framework, this becomes necessary to account for possible deficient assumptions on partitioning.

[a]The model was developed by the National Crash Analysis Center (NCAC), The George Washington University, Washington, USA.

3.1. Formulation of sub-space metamodels

Consider solving the optimisation problem (1) using metamodels. The optimisation problem becomes

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \tilde{f}_0\left(\mathbf{x}\right) \\ \text{subject to} \quad & \tilde{f}_j\left(\mathbf{x}\right) \leq 1, \qquad j = 1, \ldots, m \\ & A_i \leq x_i \leq B_i, \quad i = 1, \ldots, n \end{aligned} \tag{3}$$

where $\tilde{f}_0(\mathbf{x})$ is a metamodel of the objective function and $\tilde{f}_j(\mathbf{x})$ is a metamodel of the j-th constraint function. Given that the design variables in the optimisation problem can be categorised either as significant or insignificant for each related response. A projection can then be defined, for each response j, from the design variable space onto the space of the significant variables. This is denoted as

$$\left.\begin{aligned} \boldsymbol{\xi}_j = P_j^{\xi}\mathbf{x} \\ P_j^{\xi} : \mathbb{R}^n \mapsto \mathbb{R}^{s_j} \end{aligned}\right\}, \quad j = 0, ..., m \tag{4}$$

where n is the number of design variables in the optimisation problem and s_j is the number of significant variables for the response j. A projection onto the space of the insignificant variables is defined in the same manner as

$$\left.\begin{aligned} \boldsymbol{\psi}_j = P_j^{\psi}\mathbf{x} \\ P_j^{\psi} : \mathbb{R}^n \mapsto \mathbb{R}^{n-s_j} \end{aligned}\right\}, \quad j = 0, ..., m. \tag{5}$$

From here on the projections are described according to the following convention

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{\xi}_j \\ \boldsymbol{\psi}_j \end{bmatrix}, \quad j = 0, ..., m, \tag{6}$$

noting that the components of $\boldsymbol{\xi}_j$ and $\boldsymbol{\psi}_j$ are present in $\mathbf{x}$ in arbitrary order. The responses in the optimisation problem can then be described as

$$f_j(\mathbf{x}) = f_j\left(\begin{bmatrix} \boldsymbol{\xi}_j \\ \boldsymbol{\psi}_j \end{bmatrix}\right), \quad j = 0, ..., m \tag{7}$$

where the values of $\boldsymbol{\psi}_j$ can be chosen arbitrarily since they are deemed to be insignificant to the response. The metamodels of the responses may now be defined in the space of only the significant variables which allows a re-writing of the approximate optimisation problem according to

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \tilde{f}_0\left(\boldsymbol{\xi}_0\right) \\ \text{subject to} \quad & \tilde{f}_j\left(\boldsymbol{\xi}_j\right) \leq 1, \qquad j = 1, \ldots, m \\ & \boldsymbol{\xi}_j = P_j^{\xi}\mathbf{x}, \qquad j = 0, \ldots, m \\ & A_i \leq x_i \leq B_i, \quad i = 1, \ldots, n \end{aligned} \tag{8}$$

where each approximated response is defined only in the space of variables that are significant to the response. The optimisation problem, however, is defined in the full design variable space. This has the benefit that as each metamodel is defined only in the space of the significant variables, the sampling of training points only needs to be carried out in that space while the values of the insignificant variables are kept constant.

If the number of significant variables is small compared to the number of design variables, the density of the training points will increase leading to a better quality metamodel as compared to what would have been achieved otherwise. Note that even though there is one projection per response, practicalities may require groups of responses to use the same projection, e.g. due to several responses being evaluated from the same simulation.

3.2. Sub-space metamodels in trust regions

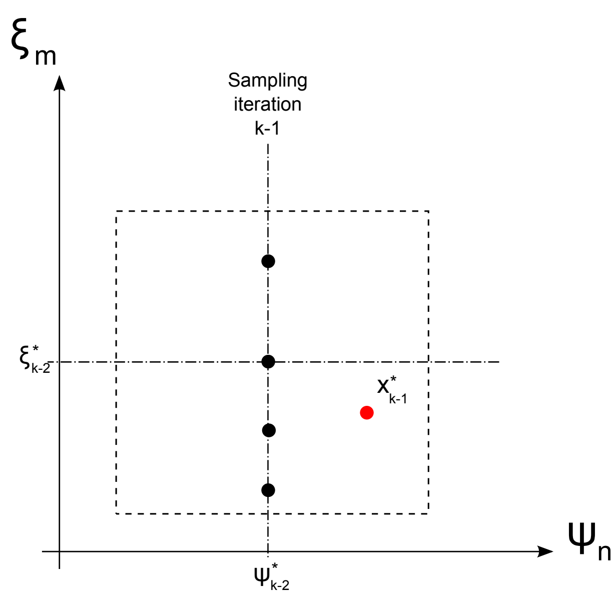
In this section an approach to building sub-space metamodels within the MAM is proposed. A recovery mechanism for erroneous assumptions for sub-space partitioning is also suggested. Sub-space metamodelse can be introduced in the MAM framework by re-writing the sequence of optimisation problems (2) as:

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & \tilde{f}_0^{\,k}\left(\boldsymbol{\xi}_0\right) \\
\text{subject to} \quad & \tilde{f}_j^{\,k}\left(\boldsymbol{\xi}_j\right) \leq 1, && j = 1, \dots, m \\
& \boldsymbol{\xi}_j = P_j^{\xi}\mathbf{x}, && j = 0, \dots, m \\
& \left.\begin{aligned} A_i^k &\leq x_i \leq B_i^k \\ A_i^k &\geq A_i \\ B_i^k &\leq B_i \end{aligned}\right\} && i = 1, \dots, n
\end{aligned}
\tag{9}
$$

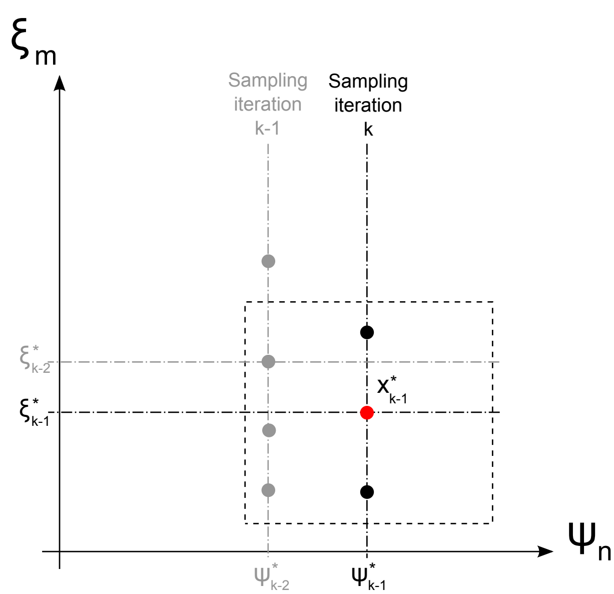
Note that the mid-range metamodels created in each iteration are here functions of the significant variables only. The significant variables for each discipline are identified by the designer. Such judgement may be based on, for instance, engineering experience or design variable ranking studies. In the previous work by [13], [14], [1] and [15], deficiencies in sub-space partitioning, i.e. by failing to identify a significant variable, can result in metamodelling errors that cannot be resolved by additional sampling. Regardless of how carefully the partitioning of variables is made, there is always a risk that significant variables will be incorrectly identified as insignificant. Therefore a recovery mechanism for such errors is needed. This can be implemented in the trust region strategy by making sure that the values of the insignificant variables for the individual response are updated at the end of each iteration according to the current best point as proposed in [2,3]. Let $\mathbf{x}_{k-1}^*$ denote the solution vector to the previous iteration $(k-1)$ for the optimisation problem (9). For each response this can be written in accordance to (7) as

$$
\mathbf{x}_{k-1}^* = \begin{bmatrix} \boldsymbol{\xi}_{k-1}^* \\ \boldsymbol{\psi}_{k-1}^* \end{bmatrix},
\tag{10}
$$

where $\boldsymbol{\xi}_{k-1}^*$ denotes the projection of the solution vector onto the space of the significant variables and $\boldsymbol{\psi}_{k-1}^*$ onto the space of insignificant variables. The values of $\boldsymbol{\psi}_{k-1}^*$ is then used as the constant values for the



(a) Sub-problem in iteration k-1



(b) Sub-problem in iteration k

Figure 4: In the sampling for iteration k the values of the insignificant variabls are kept constant at the values of the current best point, found in the previous iteration $k-1$.

insignificant variables during sampling in the current iteration, k, according to

$$\boldsymbol{\psi}_k = \boldsymbol{\psi}^*_{k-1}, \tag{11}$$

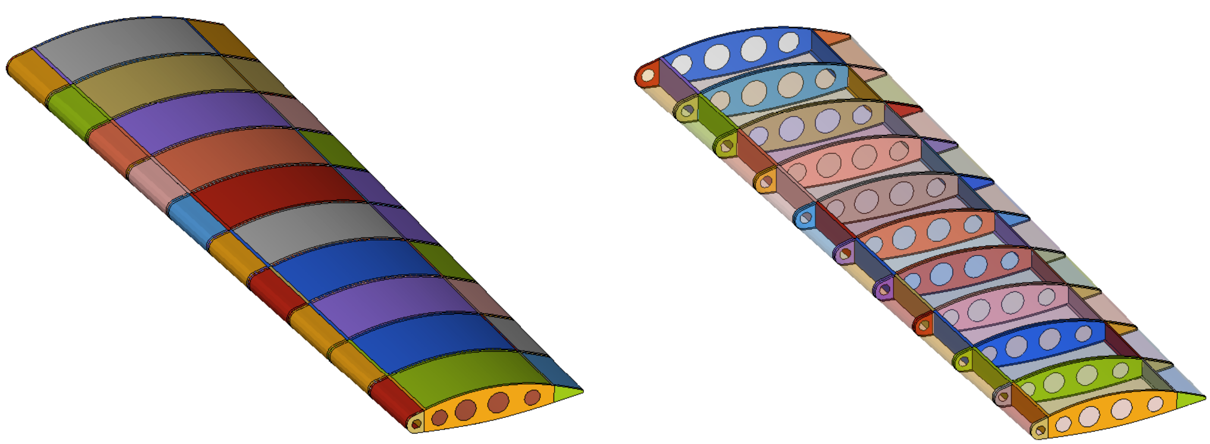
where $\boldsymbol{\psi}_k$ denotes the values of insignificant variables during sampling. Figure 4 demonstrates how the value of $\boldsymbol{\psi}_n$ changes from the previous iteration to the current for the two dimensional case. The change is due to updating the value according to the current best solution. As the metamodels for the new iteration are built using the sampling including this update, any changes in response values due to changes in insignificant variables from the previous iteration will be taken into account by the metamodels in the current iteration.

4. MDO of an aircraft wing subject to bird strike requirements

This section demonstrates a multidisciplinary optimisation of an aircraft wing structure subject to both stiffness and bird strike requirements. In order to account for all critical locations of bird impact, 10 separate bird strike simulations are considered. Advantage is taken of the local nature of the bird impact with the use of sub-space metamodels. This allows to perform the study to a much reduced computational cost than would otherwise be possible.

4.1. The wing structure

The considered wing is a 3 m long aluminium structure with a root chord of 830 mm and tip chord of 670 mm. It has two longitudinal spars and 11 ribs as shown in Figure 5. The material is precipitation-hardened aluminium (6061-T6) with properties outlined in Table 1.



(a) Complete structure.

(b) Internal structure.

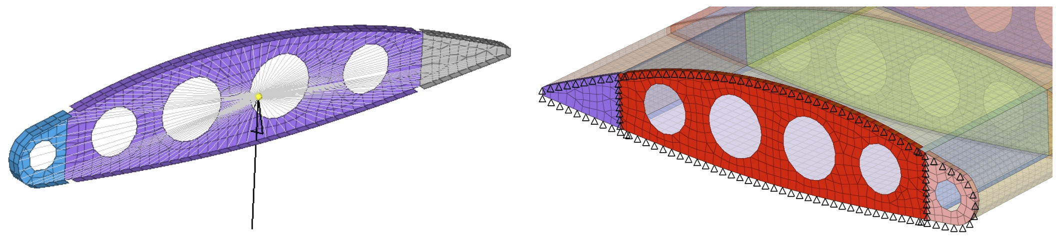
Figure 5: The wing model.

Table 1: Material characteristics for aluminium (6061-T6).

Property	Constant	Value	Unit
Material density	ρ	2.8	g/cm^3
Young's modulus	E	68.3	GPa
Poisson's ratio	E	0.33	$-$
Yield strength	σ_y	241.1	MPa
Ultimate tensile strength	σ_u	279.0	MPa

4.2. Linear static model

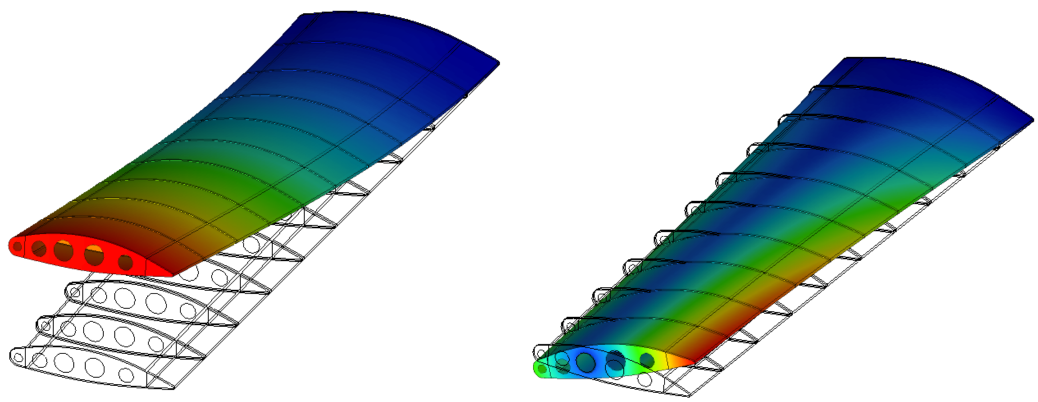
The stiffness requirements are evaluated using a linear static finite element model. The loading is applied to a single point per rib, which is then distributed to the edges of the rib using one dimensional distributing elements as shown in Figure 6a. The wing is rigidly constrained at the fuselage end of the wing in degrees of freedom 1-3 of the nodes around the edges of the rib as shown in Figure 6b.



(a) Forces/moments distributed using 1-D elements.   (b) Boundary conditions

Figure 6: Details on load application and boundary conditions.

Two load cases, shown in Figure 7, are considered. In the first one the wing is bent upwards by applying forces at each of the previously discussed rib loading points. The displacement at the tip of the wing due to the loading is considered as a response. In the second case the wing is twisted by applying moments at the rib loading points. The twist of the wing at the tip, due to the loading, is used as a response. The analysis is carried out using Altair OptiStruct [16] with the assumption of infinitesimal strain theory and isotropic linear-elastic material model. Analytical gradients can be efficiently obtained using the adjoint method.



(a) Wing bend results   (b) Wing twist results

Figure 7: Deformation of the wing for the bend and twist cases.

4.3. Dynamic explicit model

Bird strikes are high speed impact events and are thus evaluated using an explicit time-stepping scheme using Altair RADIOSS [17]. The constitutive model is elasto-plastic with isotropic hardening and failure with parameters according to Table 2. The work-hardening part of the curve is defined using tabular data of plastic strain versus stress, and the failure criterion is defined as a constant rate decrease of stress from the point of maximum tensile failure strain, ϵ_u, until reaching zero stress at the point of maximum tensile failure damage, ϵ_m. The elements are deleted as they reach the tensile strain for element deletion, ϵ_d.

The bird strike requirement is for a 4lb, or 1.81kg, bird impacting the leading edge of the wing at a speed of $150m/s$. The bird is modelled using smooth particle hydrodynamics (SPH) which is a meshless Lagrangian method based on interpolation theory. SPH is commonly used to model fluid structure interaction problems where the arbitrary Lagrangian Eulerian (ALE) formulation is expected to fail because of excessive mesh distortion. A bird exhibits fluid like behaviour at high impact speeds and can therefore be modelled realistically using the SPH formulation [18].

Table 2: Parameters of consitutive model for aluminium (6061-T6).

Property	Constant	Value	Unit
Material density	ρ	2.8	g/cm^3
Young's modulus	E	68.3	GPa
Max tensile failure strain	ϵ_u	0.08	–
Max tensile failure damage	ϵ_m	0.12	–
Tensile strain for element deletion	ϵ_d	0.13	–

The bird model, developed by Altair RADIOSS [17], has the shape of a cylinder with hemispherical ends. The radius R is 57 mm which leads to a total volume of 1939 cm^3. The model contains 41544 cells weighing approximately 0.0437 g each, adding up to a total mass of 1.81 kg and an initial density of 0.935 g/cm^3. The average distance between neighboring particles is 4.03 mm. The constitutive model is a polynomial equation of state (EOS), representing a hydrodynamic viscous fluid material defined as

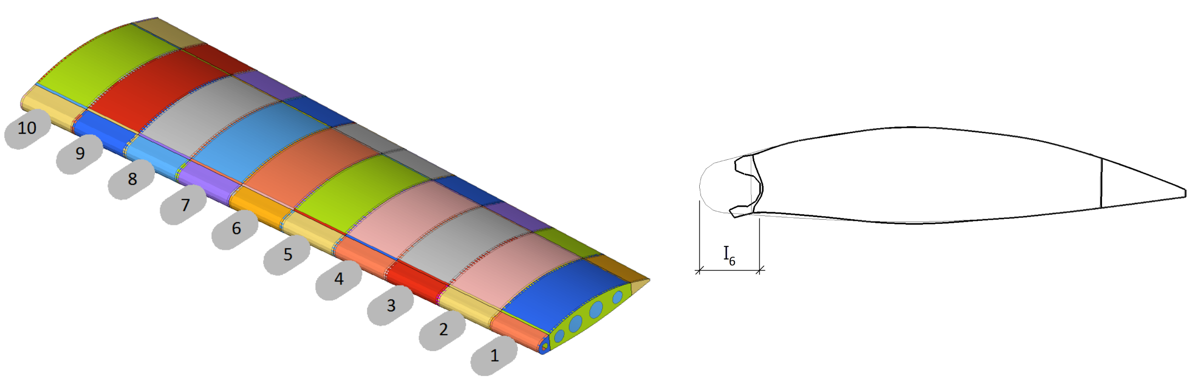
$$P = C_1 \cdot \left(\frac{\rho}{\rho_0} - 1 \right) \tag{12}$$

where P is the pressure, $C_1 = 2.106$ GPa a material constant (the bulk modulus), and ρ and ρ_0 represents the current and initial density respectively. The properties for the SPH model is summarised in Table 3.

Table 3: Parameters of consitutive model for SPH model.

Property	Constant	Value	Unit
Material density	ρ	0.935	g/cm^3
Bulk modulus	C_1	2.106	GPa
Particle mass	m_p	43.67	mg
Particle distance	h_p	4.03	mm
Number of particles	n_p	41544	-

As the impact location of the bird along the leading edge is arbitrary, several simulations need to be performed altering the impact location. To reduce the number of simulations needed, an assumption that the critical location for bird impact is in the centre of each wing section, between the ribs. This means that in total 10 simulations, with varying start point of the bird as shown in Figure 8a, are to be carried out to assess the requirements for bird strike.

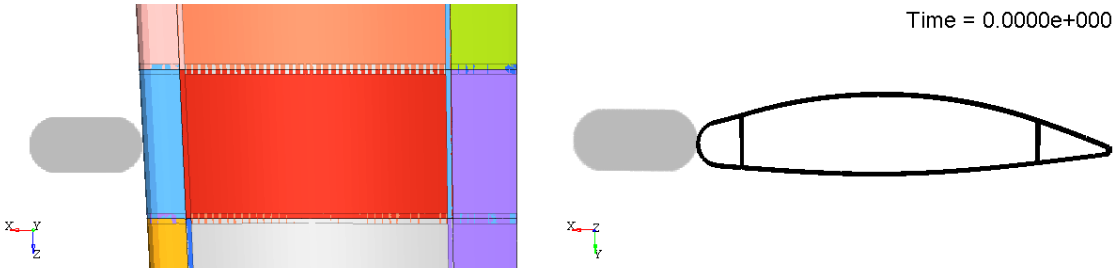


(a) Critical impact locations along the leading edge.

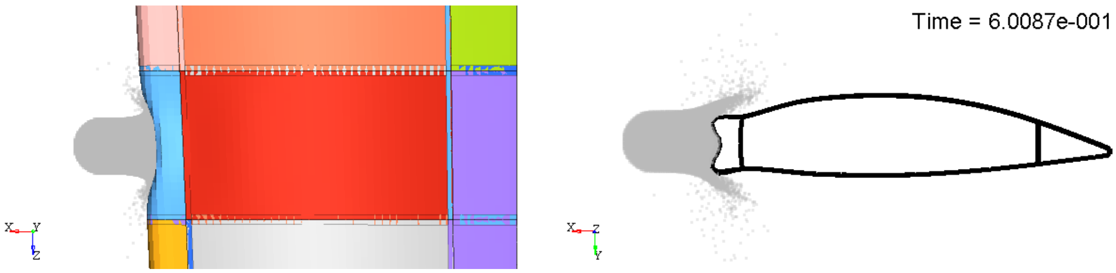
(b) Intrusion response.

Figure 8: Impact location and response definition.

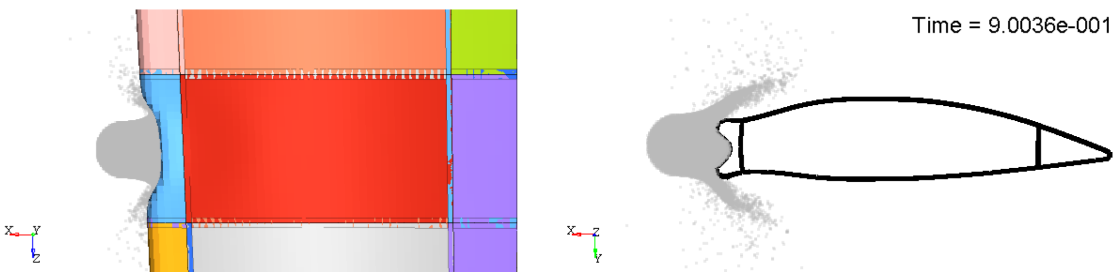
A bird strike simulation with start position 6 is shown in Figure 9.



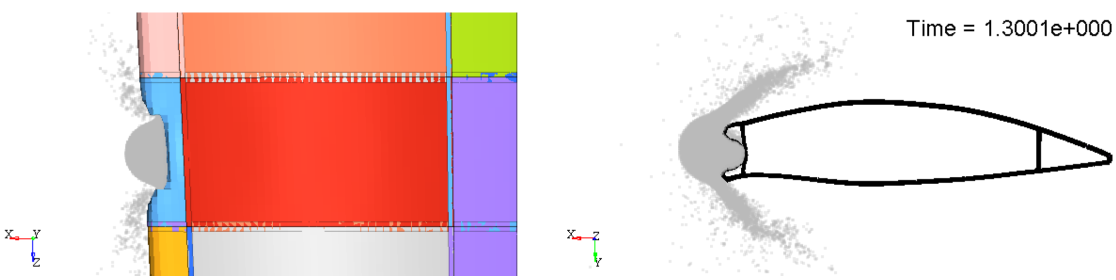
(a) t=0.0ms - Simulation start



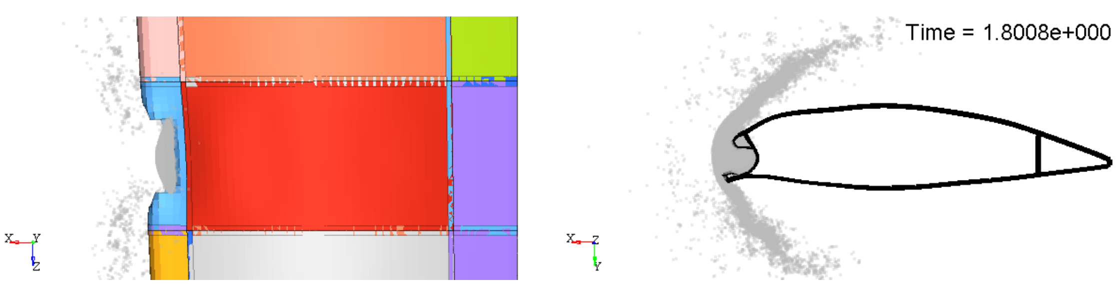
(b) t=0.6ms - Leading edge skin deformed



(c) t=0.9ms - Leading edge skin rupturing



(d) t=1.3ms - Leading edge skin contacts leading edge spar



(e) t=1.8ms - Maximum deformation leading edge spar

Figure 9: History of the bird strike simulation with starting position 6.

It shows that the bird impacts the leading edge skin which provides the initial energy absorption. The leading edge skin ruptures and makes contact with the leading edge spar which deforms as a consequence. The requirements for the impact is that the structural integrity of the wing is not to be compromised. This is interpreted such that the leading edge skin is allowed to fail, however the leading edge spar must remain intact. In this study the magnitude of intrusion into the wing, measured at the location of impact as shown in Figure 8b are used as constraints.

4.4. Optimisation procedure

The objective of the optimisation is to minimise the weight of the structure subject to meeting the previously discussed structural requirements. The design variables are the thicknesses of the 100 components. The starting thickness for all components are 3 mm with a lower bound of 2 mm and upper bound of 5 mm. Note that the leftmost rib is not designable as in this study it is constrained by boundary conditions.

The minimum number of points required by the MAM per iteration is set to the number of points needed for linear regression, $n + 1$, recalling that n is the number of design variables. It is set to this value regardless of how many points are needed by the chosen metamodel technique, in this case Kriging. It is useful to increase the number of points per iteration slightly in order to obtain better metamodels. The number of desired points per iteration is therefore chosen as $1.5n$. As the stiffness simulations have available gradients, gradient-enhanced metamodel building is used. This allows the number of points per iteration to be significantly reduced. Here the number of points required for simulations that have available gradients is chosen as $1.5n/\sqrt{n}$, resulting in 15 points per iteration. For the bird strike simulations no gradients are available which means that, for 100 design variables, the minimum number of points required is 101 while the desired number is 152. The total number of desired points for the 10 bird strike simulations would hence be 1520 points per iteration, a prohibitively large number.

In this problem sub-space metamodels can be used since the bird strike simulations have local design variable dependence. For each simulation, assumptions are made on which variables are significant to the response. For each impact location of the bird, eight design variables, shown in Figure 10 are assumed to have most of the influence on the response. Other variables may have a slight influence, and could have been considered, but for the price of an increase in computational cost. Instead, any influence from other variables will be taken care of by the recovery mechanism outlined in Section 3.2. This leads to a minimum number of 9 points and a desired number of points of 12 points per simulation and iteration. For the load case where the bird impacts the leading edge skin adjacent to the rigidly constrained rib, there are only 6 significant variables which leads to a minimum of 7 points and a desired number of 9 points. In total, a minimum of 78 and a desired number of 117 bird strike simulations per iteration.

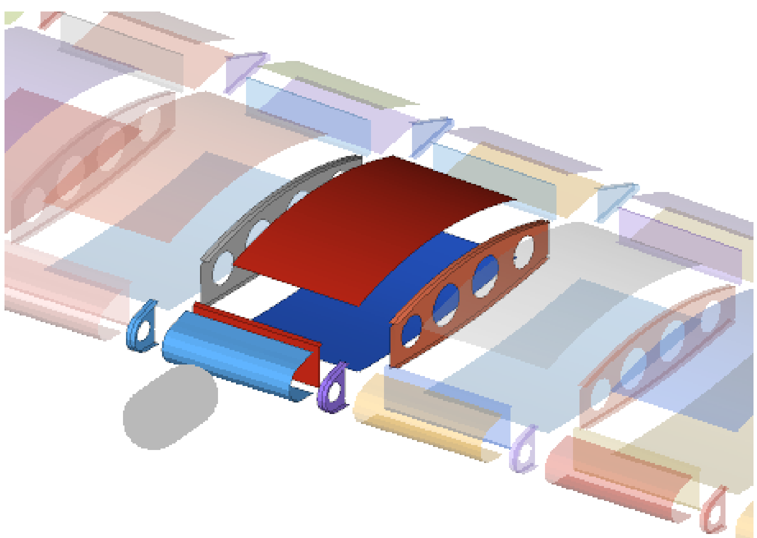
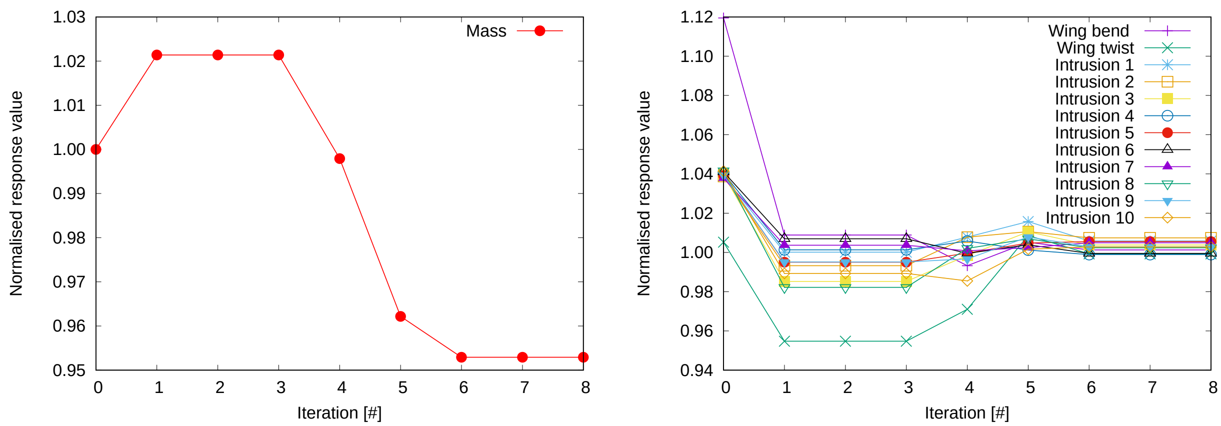


Figure 10: Assumed significant variables for bird at position 6.

4.5. Results

The history of the objective function and constraints during the optimisation are shown in shown in Figure 11. The optimisation finished in 8 iterations, having evaluated 139 stiffness simulations and 1276 bird strike simulations in total for the 10 bird locations, less than would be required per iteration had sub-space metamodels not been used. The final mass is 4.7% less than the initial design and all previously violated constraint violations were reduced to less than 1%. The initial and final response values are shown in Table 4 and the final thickness distribution is shown in Figure 12. From the result it can be noted that none of the panels have gone to the upper thickness of 5 mm, but some to the lower one of 2 mm. Many of the ribs have a resulting thickness which is in the thinner part of the thickness range. This is most likely because of the very simplistic set of static requirement used for the optimisation. As can be expected, all leading edge skins have high thickness whilst leading edge ribs are thinner. This is most likely because the leading edge skin is more likely to rupture if the leading edge rib is less compliant.



(a) Objective function.

(b) Constraint functions.

Figure 11: Optimisation history. Objective function is normalised to initial value and constraints are normalised to target.

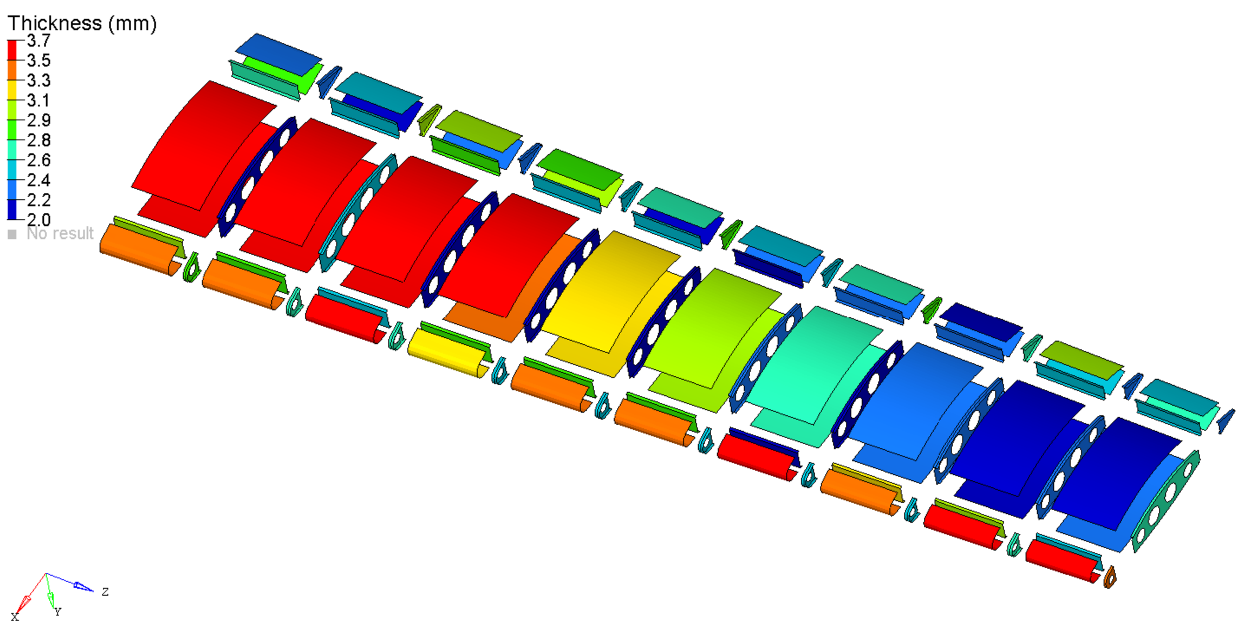


Figure 12: Final thickness for each of the considered components.

Table 4: Results of the optimisations. Objective function is normalised to initial value and constraints are normalised to target.

#	Response	Initial design	Final iteration
1	Mass		−4.7%
2	Wing bend	11.9%	0.1%
3	Wing twist	0.5%	0.0%
4	Intrusion 1	3.9%	0.6%
5	Intrusion 2	3.9%	0.7%
6	Intrusion 3	4.0%	0.4%
7	Intrusion 4	4.0%	0.0%
8	Intrusion 5	3.9%	0.6%
9	Intrusion 6	4.1%	0.0%
10	Intrusion 7	3.8%	0.5%
11	Intrusion 8	4.2%	0.2%
12	Intrusion 9	3.9%	0.3%
13	Intrusion 10	4.1%	0.3%

5. Conclusions

A multidisciplinary design optimisation of a wing structure was carried out. The considered load cases was static bending and twisting stiffness as well as bird strike requirement for impact at 10 locations. The computational cost of evaluation of the bird strike requirements is many times larger than the one of the static requirements. The optimisation was carried out using an approach previously proposed by the authors for solving MDO problems using metamodels built in individual sub-spaces of the design variable space. The approach uses existing knowledge of design variable dependence for each of the disciplines to decrease the number of required evaluations, and hence the related computational budget, in each iteration of a tust-region based optimisation procedure. The optimisation finished in 8 iterations having evaluated 139 stiffness simulations and 1276 bird strike simulations in total, less than would be required per iteration had sub-space metamodels not been used. The final result is a mass save of 4.7% and a reduction of all previously violated constraints to less than 1%.

Acknowledgments

The authors greatly acknowledges funding from European Union Seventh Framework Programme FP7-PEOPLE-2012-ITN under grant agreement 316394, Aerospace Multidisciplinarity Enabling DEsign Optimization (AMEDEO) Marie Curie Initial Training Network.

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