Aerodynamic Shape Optimization Using the Adjoint-based Truncated Newton Method

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This paper presents the development and application of the truncated Newton (TN) method in aerodynamic shape optimization problems. The method has been developed in OpenFOAM for incompressible laminar flows with the aim of showing its advantages over standard gradient-based optimization algorithms. Generalized minimal residual (GMRES) is used for solving the Newton equations. GMRES only requires the computation of the product of the Hessian of the objective function and a vector which enables us to refrain from computation of the Hessian itself; computational cost of the latter scales with the number of design variables and becomes unaffordable in large-scale problems with numerous design variables. A combination of the continuous adjoint method and direct differentiation is used for computation of all Hessian-vector products. Also, a grid displacement PDE (Laplace equation) is used for computing necessary derivatives of grid displacement w.r.t the design variables. The optimization of a 3D S-bend duct provided as one of the test cases of the AboutFLOW ITN programme is presented with the aim of minimizing the total pressure losses.

I. Introduction

In aerodynamic shape optimization, the adjoint approach is widely used for computation of the objective function gradient w.r.t the design variables since the computational cost is independent of the number of the design variables. Moreover, Newton optimization algorithms might lead to faster convergence (at least, in terms of optimization cycles) compared to conventional optimization algorithms solely relying on the gradient. However, the computational cost of the Hessian matrix required for Newton method scales with the number of design variables which is the main reason that the exact Newton method is limited to applications with few design parameters. One alternative could be the exact initialization of the quasi Newton method in which the exact Hessian is only computed in the first optimization cycle and then, it is approximately updated[1]. Although this method could be more efficient than the Newton method, computation of the exact Hessian, even for once, might become prohibitive in large cases with numerous design variables. In these cases, TN can be used instead.

II. Development of the TN method

Assuming F is a general objective function, the Newton method accelerates the optimization algorithm in which the design variables b_i (i = 1,…,N) are updated by solving the Newton's system :

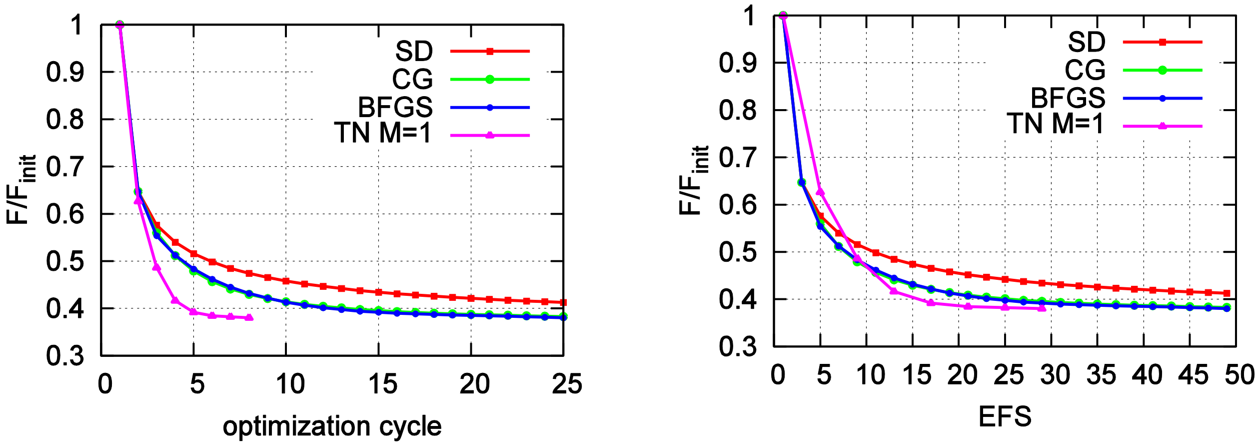
$$\frac{\delta^2 F}{\delta b_i \delta b_j} \Delta b_j = -\frac{\delta F}{\delta b_i} \qquad (1)$$

$$b_j^{new} = b_j^{old} + \Delta b_j \;\; j = 1, \dots, N$$

The r.h.s of equation (1) can be computed via the adjoint method since it is nothing more than the gradient of F. Although, in theory, Hessian is symmetric and any iterative algorithm which only relies on the computation of the Hessian-vector products without requiring the knowledge of the Hessian itself should be appropriate for solving equation (1), this is not the case in CFD applications. The Hessian expression which is obtained through the AV-DD approach (i.e., use of adjoint method for computation of $\frac{\delta F}{\delta b_i}$ and direct differentiation for computation of variations of the adjoint and primal fields) becomes symmetric if all equations are converged to machine accuracy[2]. In CFD-based optimizations, it is quite common not to converge primal and adjoint equations to machine accuracy in each optimization cycles in order to reduce the total CPU cost. This can deteriorate the symmetry of the Hessian matrix which renders algorithms like conjugate gradient (CG) inappropriate for solving the equation (1). Hence, GMRES which is for non-symmetric systems is used instead.

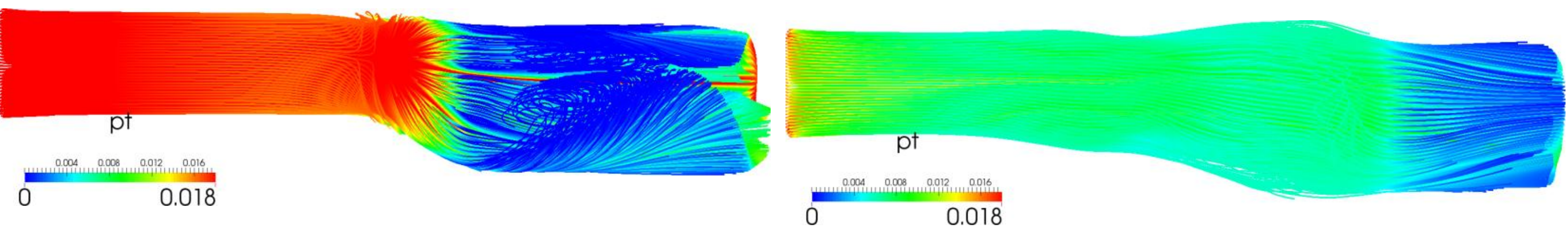
III. Results

In this section, the results of optimizing a 3D S-bend duct with the aim of minimizing the total pressure losses are presented. The flow is laminar with a Reynolds number of Re = 400 based on the inlet hydraulic diameter and the mesh comprises 474000 hexahedra. A 9 × 7 × 9 control grid is used to parameterize part of the duct which results in 375 design variables. In figure 1, the convergence history of the developed TN algorithm is compared to those of steepest descent (SD), conjugate gradient (CG) and BFGS methods. It can be observed that TN outperforms other methods in terms of both the number of optimization cycles and the equivalent flow solutions (EFS). In figure 2, the flow streamlines on the reference and optimized shape are compared indicating the significant reduction of the flow recirculation which in turn leads to a 60% decrease in the total pressure losses.



(a) **(b)**

Figure 1. Comparison of the convergence of SD, CG, BFGS and TN w.r.t (a) optimization cycles (b) EFS



(a) **(b)**

Figure 2. Velocity stream lines plotted for (a) reference shape, (b) optimized shape. Streamlines are colored based on the total pressure values. The intense flow recirculation close to the bottom side of the wall has drastically been reduced leading to a 60% decrease in the objective function.

References

[1]Zervogiannis, T., Papadimitriou, DI., and Giannakoglou, KC., "Total Pressure Losses Minimization in Turbomachinery Cascades Using the Exact Hessian," *Computer Methods in Applied Mechanic and Engineering*, Vol. 199, No. 41, 2010, pp. 2697, 2708.