Probabilistic analysis of topologically optimized structures considering geometric imperfections

Julian K. Lüdeker* and Benedikt Kriegesmann†
Hamburg University of Technology, Am Schwarzenberg-Campus 4, 21073 Hamburg, Germany

I.Introduction

THE industrialization of additive layer manufacturing has raised the intention of aircraft industry for topology optimization, as it allows manufacturing arbitrary shapes of structures. Standard topology optimization using the SIMP approach minimizes the compliance for a given volume fraction.[1] Several works enhanced topology optimization for considering stress constraints,[2,3] as this is more relevant for practical application that the stiffness. However, these approaches do not consider the sensitivity with respect to uncertainties and therefore provide designs with little robustness.

Kharmanda et al. suggested combining reliability based design optimization with topology optimization, which is referred to as reliability-based topology optimization (RBTO).[4] Other studies followed this idea and confirmed the finding that the design obtained from RBTO can differ significantly from the one obtained using classical topology optimization.[5,6] In these publications only parameters were considered as random which are independent of the design, such as Young's modulus, load magnitude and the size of the design space. The scatter of important measured like material strength, defects and geometry was not considered. Especially geometrical deviations are challenging to be considered as random within the RBTO, but it is known to have an effect on stability and fatigue performance.

The present work suggests a procedure to account for geometric deviations from the nominal structure in probabilistic analyses. First, a procedure is presented to describe the result of a topology optimization by Non-Uniform Rational B-Splines (NURBS). Secondly, the utilization of the NURBS representation for probabilistic analyses is demonstrated.

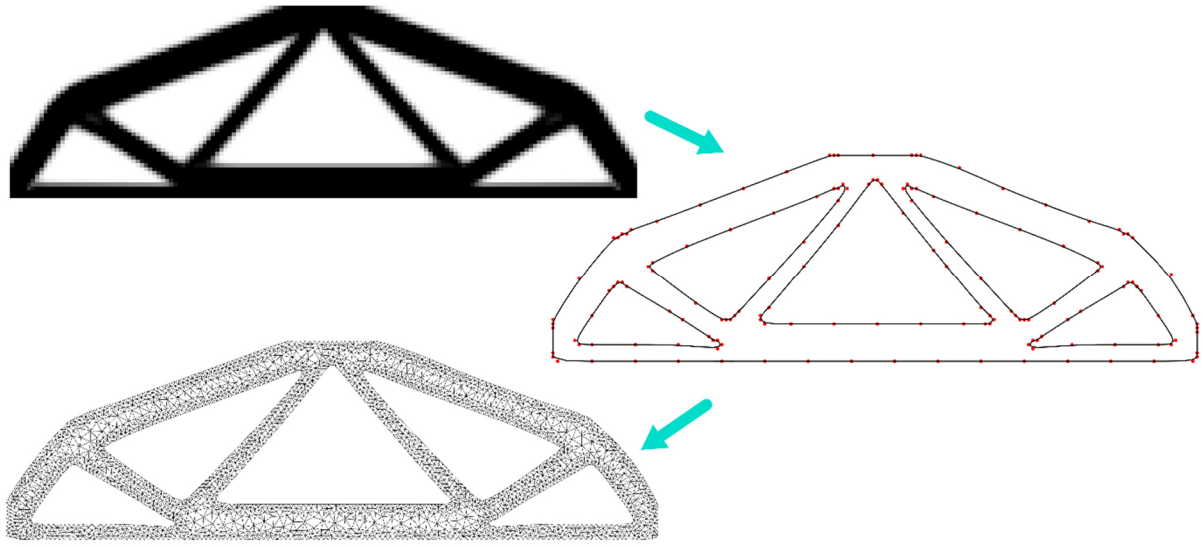

Figure 1. Result of a topology optimization (top), an automatically generated geometric interpretation (middle) and the finite element mesh (bottom).

II.Parameterization of Topologically Optimized Geometry

Topology optimization using the SIMP approach provides a distribution of normalized densities, which vary between 0 (no material) and 1 (fully solid). Densities in between 0 and 1 can be interpreted as the homogenized property of a somehow porous cell. However, in practice often a choice is made to consider or not a cell as part of the structure by choosing a density threshold, especially since areas with densities in between 0 and 1 are located at the edge of the final structure (see Figure 1, top).

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* Scientific assistant, Working group New Design and Sizing Methods for Hybrid Airframe Structures.
† Juniorprofessor, Working group New Design and Sizing Methods for Hybrid Airframe Structures.

For the current work, the objective was to define the edge independent of an engineer's choice. This is achieved by defining the edge of the structure along the lines of maximal gradients of the density fields obtained from topology optimization. The representation of the edge shall be parameterized such that its parameters can be used for further shape optimization and probabilistic analyses. Therefore, NURBS have been used to describe the structure edges (see Figure 1, middle), which allows adjusting the geometry by modification of the control points. (For details on NURBS, see e.g. Ref. [7].) The control points of NURBS (red dots in Figure 1, middle) are concentrated at corners of the geometry. From the NURBS representation, a Finite Element model is generated (see Figure 1, bottom), which is analyzed with respect to stress distribution.

III.Probabilistic analysis

A probabilistic analysis is performed considering geometric imperfection as random input and the maximal occurring stress as output parameter. The scatter of geometry is described by the coordinates of the control points of the NURBS $x_{c,i}$ and $y_{c,i}$, which are summarized in the random vector $\mathbf{X}$.

$$\mathbf{x} = \left(x_{c,1}, y_{c,1}, \ldots, x_{c,i}, y_{c,i}, \ldots, x_{c,p}, y_{c,p} \right)^T \tag{1}$$

Given a sample of manufactured specimens, the control point coordinates are adjusted to describe the geometry of the specimen. Hence, a realization $\mathbf{x}^{(j)}$ of the random vector $\mathbf{X}$ is obtained for each specimen. From the realizations, the mean vector $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$ of random vector $\mathbf{X}$ are estimated. These are used for transforming the random vector $\mathbf{X}$ to the random vector $\mathbf{Z}$, by

$$\mathbf{X} = \mathbf{Q}\,\mathbf{D}^{\frac{1}{2}}\,\mathbf{Z} + \boldsymbol{\mu} \qquad \boldsymbol{\Sigma} = \mathbf{Q}\,\mathbf{D}\,\mathbf{Q}^T \ . \tag{2}$$

The matrixes $\mathbf{Q}$ and $\mathbf{D}$ are obtained from the spectral decomposition of $\boldsymbol{\Sigma}$. The random vector $\mathbf{Z}$ has uncorrelated entries with a mean value of 0 and a standard deviation of 1. Furthermore, the length of $\mathbf{Z}$ cannot exceed the number of measurements used to build the covariance matrix. Therefore, this transformation significantly reduces the number of parameters required to describe geometric imperfections. (For more details, see e.g. Ref. [8].) In the present case, $p = 158$ control points are used and $\mathbf{X}$ has a length of 316. Since 20 specimens are considered to build $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, $\mathbf{Z}$ has a length of only 19.

The random vector $\mathbf{Z}$ is considered to generate samples in a Monte Carlo simulation as well as for preforming first-order second-moment analysis,[8] which requires estimating the gradient of the probabilistic objective function (here, maximum stress) with respect to scattering parameters. The combination of the NURBS representation with the transformation (2) allows describing the scatter of a complex shape with very few parameters and therefore allows for very efficient probabilistic analysis.

For the present work, no measurements of manufactured specimen are available. Therefore, virtual specimens have been generated by perturbation the initial geometry. The subsequent steps are the same, but an experimental validation of the procedure is pending.

IV.Conclusion and Outlook

An efficient procedure is presented to capture the randomness of geometric imperfections for probabilistic analyses of topologically optimized structures. The procedure is easy to integrate into a reliability based shape optimization of complex structures.

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