On the use of the unsteady continuous adjoint method for the optimization of jet-based flow control systems

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Summary

This paper is concerned with the use of the unsteady continuous adjoint method for optimizing an active flow control system based on pulsating jets. The amplitudes, time-phases, period and locations of the jets are used as design variables. Without loss in generality, the objective functions this paper is dealing with are the time-averaged squared lift and drag. Whenever period is among the designing variables, the terms resulting from the application of the Leibniz theorem in time are carefully taken into account in the derivation of the expression for the sensitivity derivatives. Their contribution together with the role of the bounds of the time-integration are investigated. The flow around a cylinder together with a less costly mathematical problem, sharing the same features with the flow control problem, are presented.
Keywords: *Unsteady continuous adjoint method, active flow control, pulsating jets*

1 Introduction

Active flow control is an effective way to control the boundary layer development so as to avoid or delay separation. In literature, pulsating or synthetic jets have been used to control internal and external flows[1]. The optimal values of the actuation parameters can be computed using a gradient-based optimization method supported by the continuous or discrete adjoint method.

This paper is dealing with the unsteady continuous adjoint method[2], in which the adjoint equations and the sensitivity derivatives are derived by differentiating the objective function augmented by the field and time integral of the product of the flow (state) equations and the adjoint variables. The adjoint equations are discretized and numerically solved to compute the time-varying adjoint fields and, then, the sensitivity derivatives.

This paper focuses on the development of the unsteady continuous adjoint for the optimization of the flow control around a cylinder, though findings can be generalized to industrial cases. Compared to the existing literature, the studied cases involve the jets period and locations as additional design variables.

In previous work by the authors[3], the optimal locations of the jets were selected, according to the computed sensitivity map, at the boundary areas with the highest potential for improvement, without running an optimization loop for them. In this paper, the jets locations are among the design variables and their values are updated at each optimization cycle. This is why a new expression for the jets velocities, which can be differentiated w.r.t. their location, is used.

Having the jets period among the design variable requires a careful treatment. When the jets act on the cylinder, the flow period becomes equal to the jets period. In the continuous adjoint formulation, if the jets period is to be optimized, the Leibniz theorem is applied leading to terms depending on the limits of the time integral contributing to the objective function. A term-by-term analysis that helps the reader understand the impact of these terms on the gradient and whether the latter depends on the starting instant of the integration, is carried out. The optimization cases aim at minimizing mean squared drag or lift, with various combinations of the amplitudes, phases, period and/or locations of jets as design variables. Since, for a thorough analysis, we have good reasons to keep the costs as low as possible, a representative mathematical problem was solved too; this problem sheds light to the role and numerical implication of all terms appearing in the sensitivity derivative expression.

2 Flow model

The flow is laminar, modeled by the Navier-Stokes equations for incompressible flows, namely

$$R^p = -\frac{\partial v_i}{\partial x_i} = 0 \qquad (1)$$

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$$R_i^v = \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right] = 0 \tag{2}$$

where v_i , $i = 1,2(3)$ and p stand for the velocity components and the static pressure divided by the density, respectively and ν is the bulk viscocity. Repeated indices imply summation, unless stated otherwise.

2.1 Velocity expression for fixed jets

The velocity vectors along the body surface Γ_w are either zero (solid wall) or equal to those of the imposed jets. In the controlled case, with superscript k denoting points where pulsating jets are applied, the corresponding jet velocity components are

$$v_i^k = A^k \left[sin\left(\frac{2\pi}{T_{jets}}(t - t_0^k)\right) - 1\right] n_i,\ i = 1,2(3) \tag{3}$$

where A^k is the amplitude and t_0^k is the time-phase of the k^{th} jet velocity, T_{jets} is the common for all jets period and n_i are the components of the unit vector normal to the boundary. The direction of $\vec{n}$ is such that a positive amplitude A^k corresponds to blowing whereas a negative one to suction.

2.2 An alternative expression for jets velocities

In case jet locations are among the design variables a different way of expressing the jet velocity must be used. So their amplitudes are defined as a function of the angular locations along the cylinder circumference ϕ of their centers, namely

$$v_i(\phi) = A^\star(\phi)\left[sin\left(\frac{2\pi(t-t_0)}{T_{jets}}\right) - 1\right] n_i,\ \ i = 1,2(3) \tag{4}$$

where $0 \leqslant \phi \leqslant \pi$, since only the top-half cylinder circumference is controlled and, then, mirrored to the bottom-half, and

$$A^\star(\phi) = \sum_{k=1}^{N_{jets}} A^k e^{\beta^k(\phi)}\ ,\ \beta^k(\phi) = -\frac{\left(\phi - \phi^k\right)^2}{z} \tag{5}$$

All jets act in the normal to the surface direction. In eq. 5, ϕ^k is the angular location of each jet along the cylinder circumference and z is a constant related to the angular width of each jet slot. According to eq. 4, the fluid velocity on the cylinder circumference is equal to either zero (solid wall) or the local jet velocity, depending on the ϕ value. The great advantage of eq. 4, compared to eq. 3, is that the former is differentiable w.r.t. to ϕ^k.

3 Flow Control Optimization using Continuous Adjoint

This paper is dealing with the minimization of (the half of) the time-averaged squared lift or drag. Both objectives are cast in the form of the same objective function

$$F = \frac{1}{2T}\int_{\breve{a}}^{\breve{a}+T} g^2\, dt \tag{6}$$

where

$$g = \int_{\Gamma_w}\left[-\nu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) n_j + pn_i - \left|v_j n_j\right| v_i\right] r_i d\Gamma \tag{7}$$

Here, r_i are the components of a unit vector aligned with (drag) or being normal to (lift) the farfield velocity vector and $\breve{a}$ is the user-defined starting instant of the time integration in eq. 6. In eq. 7, the last term expresses the momentum effect of jets acting on the cylinder. In periodic flows, the objective function value and the corresponding sensitivity derivatives should be independent of $\breve{a}$; among other, this paper investigates the role of the $\breve{a}$ value in the gradient of F, computed by the continuous adjoint method, and presents some interesting findings.

The augmented objective function L is defined as

$$L = F + \int_T\int_\Omega u_i R_i^v d\Omega dt + \int_T\int_\Omega q R^p d\Omega dt \tag{8}$$

where u_i and q are the adjoint to the v_i and p fields, respectively. Next step is to compute the variation in L w.r.t. the design variables, namely A^k, t_0^k and T_{jets}. Hereafter, symbol $\int_T$ stands for $\int_{\breve{a}}^{\breve{a}+T}$.

Unless jets intensity is negligible, the flow period T is determined by the jets period, $T = T_{jets}$. This is why, in what follows, symbol T is used instead of T_{jets}. In eq. 6, the upper bound of the time integration is a function of T and, therefore, the Leibniz theorem must be applied for the differentiation of L, if T is considered as one of the design variables b_n.

The variation of L w.r.t. b_n becomes

$$\begin{aligned}\frac{\delta L}{\delta b_n} &= \frac{1}{T}\int_T g\frac{\partial g}{\partial b_n}dt + \left[\frac{1}{2T}g^2\Big|_{t=\breve{a}+T} - \frac{1}{2T^2}\int_T g^2 dt\right]\frac{\delta T}{\delta b_n} \\ &+ \int_T\int_\Omega u_i\frac{\partial R_i^v}{\partial b_n}d\Omega dt + \int_T\int_\Omega q\frac{\partial R^p}{\partial b_n}d\Omega dt\end{aligned} \tag{9}$$

The unsteady field adjoint equations[4] result by making the coefficients of $\frac{\partial v_i}{\partial b_n}$ (adjoint momentum equations) and $\frac{\partial p}{\partial b_n}$ (adjoint continuity equation), in the field integrals of eq. 9, equal to zero and yield

$$-\frac{\partial u_i}{\partial x_i} = 0 \tag{10}$$

$$-\frac{\partial u_i}{\partial t} - v_j\frac{\partial u_i}{\partial x_j} + u_j\frac{\partial v_j}{\partial x_i} - \frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + \frac{\partial q}{\partial x_j} = 0 \tag{11}$$

None of eqs. 10 and 11 is affected by the selected objective function, since F is free of field integrals.

After satisfying the field adjoint equations, the derivatives of F w.r.t. the control variables b_n yields

$$\frac{\delta F}{\delta b_n} = \underbrace{\int_{\breve{a}}^{\breve{a}+T}\int_{\Gamma_w} Q_{1,i}\frac{\partial v_i}{\partial b_n}d\Gamma dt}_{G_1} + [G_2+G_3+G_4]\frac{\delta T}{\delta b_n} \tag{12}$$

where

$$Q_{1,i} = u_i v_j n_j + \nu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)n_j - qn_i \tag{13}$$

$$+u_i\left|v_j n_j\right| + u_j v_j \frac{v_m n_m}{|v_r n_r|}n_i \tag{14}$$

and

$$G_2 = \left.\frac{1}{2T}g^2\right|_{t=\breve{a}+T}$$
$$G_3 = -\frac{1}{2T^2}\int_T g^2 dt$$
$$G_4 = -\left.\int_\Omega u_i \frac{\partial v_i}{\partial t}d\Omega\right|_{t=\breve{a}+T}$$

The last three G terms are derived from the application of the Leibniz theorem and appear only if T is unknown. $\frac{\delta F}{\delta b_n}$ is normally expected to be independent of the $\breve{a}$ value. The question is whether this is also true from the numerical point of view. Let us though further investigate the terms appearing on the r.h.s. of eq. 12. Since amplitudes A^k and phases t_0^k are independent of T, terms including $\frac{\delta T}{\delta b_n}$ in the expressions for $\frac{\delta F}{\delta A^k}$ and $\frac{\delta F}{\delta t_0^k}$ are zero and the remaining integral term (G_1) takes on a value which does not depend on $\breve{a}$, due to the periodicity of the involved quantities. In contrast, this is not evident in the $\frac{\delta F}{\delta T}$ formula (for $\frac{\delta T}{\delta b_n}=1$). It can be shown that $\frac{\partial v_i^k}{\partial T}$ oscillates with period equal to T and amplitude depending linearly on t. This behaviour reflects upon the integrand $Q_{1,i}\frac{\partial v_i}{\partial T}$ and the corresponding integral is periodic w.r.t. $\breve{a}$. According to their definition, terms G_2 and G_4 which contribute to $\frac{\delta F}{\delta T}$ are affected by the choice of $\breve{a}$, since they are both expressed at $t=\breve{a}+T$. Therefore, $\frac{\delta F}{\delta T}$ is the synthesis of the periodic, in terms of $\breve{a}$, terms G_1, G_2 and G_4, which counterbalance each other and lead to a constant value, independent of $\breve{a}$ (fig. 1). This term-by-term investigation is presented in both the jet-based flow control optimization around a cylinder and the 1D mathematical problem. For the latter, closed-form expressions are derived.

The differentiation of the jet velocity components v_i^k w.r.t. b_n for both jet velocity profiles, eqs. 3 and 4 can easily be derived. It is worth mentioning though that, based on eq. 4, where the jet location also varies, inequality constraints must be imposed, in order to have distinct jet slots and avoid overlapping. The constraint functions are

$$c_1 = \phi_1 + \frac{w}{2} < 0$$
$$c_i = \phi_{i-1} - \phi_i + w < 0\ ,\ i\in[2,N_{jets}]$$
$$c_{N_{jets}+1} = \pi - \phi_{N_{jets}} + \frac{w}{2} < 0 \tag{15}$$

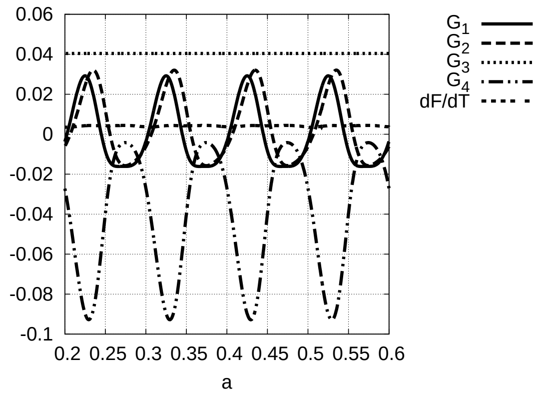

Figure 1: Computed G_i terms and the objective function gradient w.r.t. period T, in terms of $\breve{a}$ (marked as a on the x axis).

where w is the angular width of each jet slot, determined by the constant z. Constraints c_1 and $c_{N_{jets}+1}$ ensure that the first and last jet won't exceed 0 and π rads respectively. By satisfying the c_i constraint, $i\in[2,N_{jets}]$, the ith jet will be located between its adjacent jets, without overlapping them.

These constraints are imposed via the inner barrier method,[5] according to which the objective function takes the form

$$F_{constr} = F - \omega_p \sum_{i=1}^{N_{jets}+1}\frac{1}{c_i} \tag{16}$$

where ω_p is a weight which is initialized at a relatively large positive number and decreases in the course of the optimization.

4 Applications

The presented adjoint method is applied to the minimization of the time-averaged squared lift or drag, for the flow around a cylinder at Re = 100. In all cases, 20 pulsating jets are placed along the circumference of the cylinder with the angular width of each jet slot being constant and equal to $\frac{\pi}{120}D$, where D is the cylinder diameter. In some of the cases, the jet locations are fixed and equidistributed along the cylinder circumference (in these cases, the velocity profile is given by eq. 3) and in some other the jet locations are free to vary (eq. 4).

In the uncontrolled case, the flow is characterized by the generation of von Karman vortices with period $T\approx 0.59s$. In the controlled cases, only the jets along the top-half of the cylinder perimeter are controlled and their values are mirrored to the jets along the bottom-half. For all controlled cases, other than those in which the jets period is unknown, the jets operate with fixed period equal to $T=\frac{D}{U_\infty}=0.1s$.

Details about the optimization results, as well as the computed primal and adjoint flow fields will be included in the full paper. In this extended abstract, three figures, regarding the flow control around the cylinder, are presented (figs.2,3,4); Their captions clearly explain and comment on

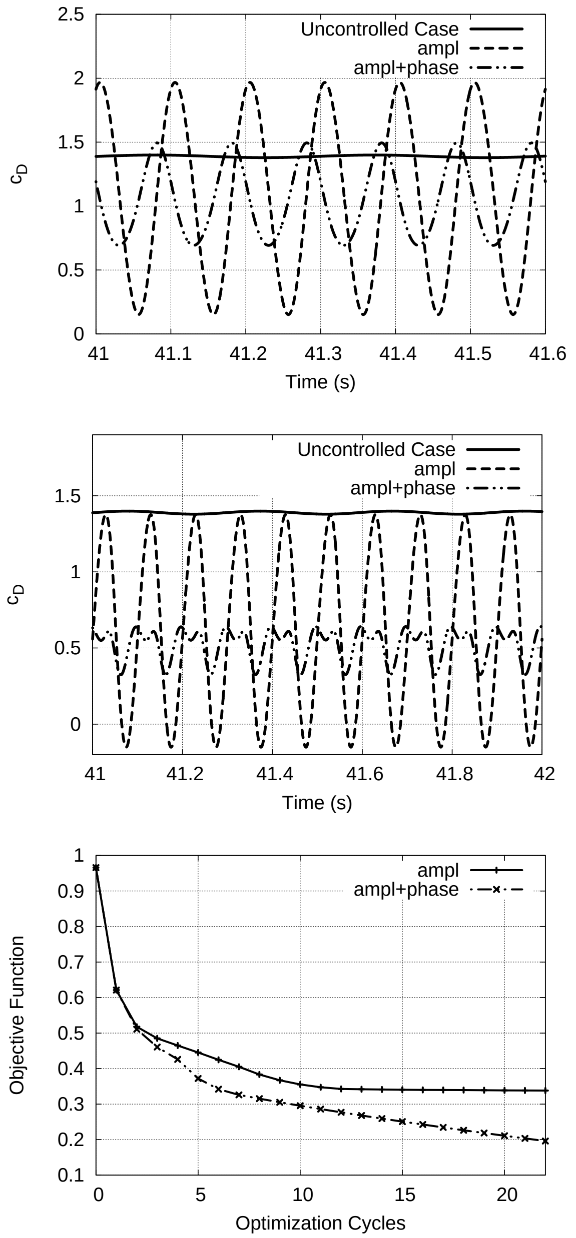

Figure 2: Comparison of the drag coefficient for the cases of lift (top) and drag (middle) minimization. For each of the objective forces, two sets of design variables were used. In the first one the jets amplitudes are allowed to change with fixed phases (equal to 0*s*), while in the second one both amplitudes and phases are allowed to vary. Lift acting on the cylinder is eliminated in all cases, since the presence of jets eliminates vortex shedding. In all cases, period was fixed and equal to 0.1*s*. Optimization (steepest descent) algorithm convergence for minimizing drag (bottom). It is seen that, when both jets amplitudes and phases are allowed to change the optimization, apart from leading to a better optimal value, also converges faster. Moreover, in this case, the drag oscillation amplitude is greatly decreased.

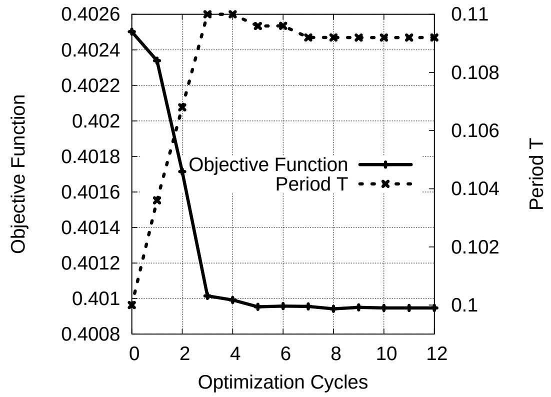

Figure 3: Convergence of the optimization algorithm for minimizing drag, with period T as the only design variable. This optimization is a continuation of a previous test case which used the amplitudes as design variables. In the present study, the latter are kept fixed and a further reduction of the objective function is seeked by varying only the period. Computed objective function and T values at each optimization cycle are presented.

the presented results. The mathematical 1D case will be presented in the full paper.

References

[1] Zheng, X., Hou, A., and Zhou, S. Control of unsteady separated flows inside axial compressors by synthetic jets. In *43th AIAA Aerospace Sciences Meeting and Exhibit* (, Reno, Nevada, 2005).

[2] Jameson, A. Aerodynamic design via control theory. *Journal of Scientific Computing* **3**, 233–260 (1988).

[3] Zymaris, A., Papadimitriou, D., Papoutsis-Kiachagias, E., Giannakoglou, K., and Othmer, C. The continuous adjoint method as a guide for the design of flow control systems based on jets. *Engineering Computations* **30**(4) (2013).

[4] Zymaris, A., Papadimitriou, D., Giannakoglou, K., and Othmer, C. Continuous adjoint approach to the Spalart-Allmaras turbulence model for incompressible flows. *Computers & Fluids* **38**(8), 1528–1538 (2009).

[5] Nocedal, J. and Wright, S. *Numerical Optimization.* Springer, (1999).

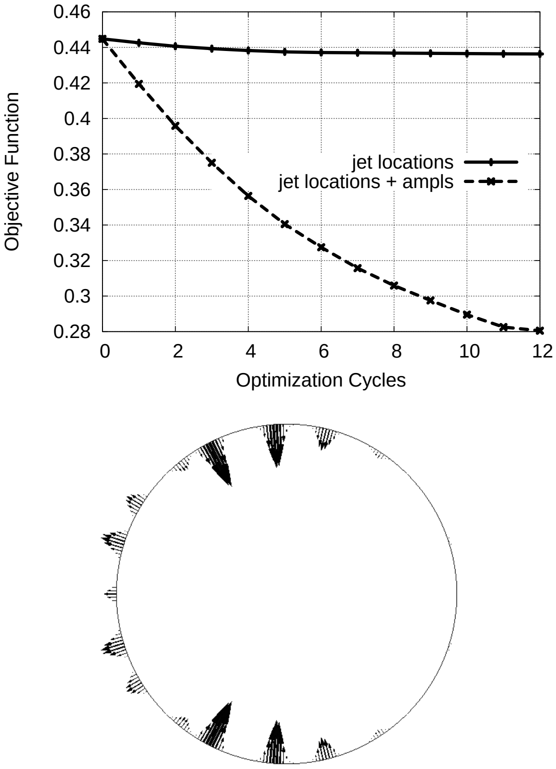

Figure 4: Top: Drag minimization convergence with design variables either the jets locations or both jets locations and amplitudes. This optimization is a continuation of a previous test case, where amplitudes had been the design variables. Bottom: The optimal location of the jets is also presented for the first case. It is seen that the jets should be applied in the front half of the cylinder so as to prevent the flow from separating. The most downwind jets in the separated region, have a really small amplitude and play a minor role in the optimization process. In both cases, phase and period were kept constant ($0s$ and $0.1s$, respectively).